FAULT ISOLATION USING FREQUENCY RESPONSE TECHNIQUES

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THESIS

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Fault Isolation Using Frequency

Response Techniques

by

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ABSTRACT

An investigation of using the response of a circuit at selected test frequencies to isolate faulty circuit components is made. A procedure using a sensitivity approach for intelligent selection of test frequencies is developed. The developed procedure is tested and the results are compared with results using conventional procedures. The effect of random, within tolerance variations of nonfaulty components on the results is studied for both conventional and developed procedures.

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I. INTRODUCTION

Maintenance of electronic equipment is a problem that increases as equipment becomes more complex. With ever increasing costs (of equipment) it also becomes imperative to decrease equipment downtime as much as possible. To this end a fast, reliable method of locating circuit failures must be used.

It has been customary to isolate failures by testing individual components. This is usually done by isolating the fault to a particular stage, followed by subassembly isolation and finally component isolation by individual testing.

With the availability and flexibility of digital computers, a possible solution to the problem of excessive downtime is to use a computer to help isolate the fault. The first step in accomplishing this is the development of a testing and isolation procedure which could be programmed for the digital computer.

A recently presented procedure has been examined by a number of individuals [1], [2], [3], and [4]. The procedure was described by Seshu and Waxman [1] with other investigators testing and making applications of the procedure with few modifications.

The basic procedure is to examine the response of a circuit under test at selected frequencies. Comparisons of the results with a list of "library results" can pinpoint a faulty component.

Most investigators have used test frequencies near the poles and zeros of the function under consideration. Other frequencies have been chosen between the pole and zero frequencies [2] and [4]. In this study an alternate method of test frequency selection was developed and tested and results were compared with results using other methods.

The procedures for signature comparison used by other investigators have not been explained in detail in most reports. However, it appears that a nearest neighbor approach is the most common technique used to date. Two alternate methods were examined and are discussed with conclusions as to their practicality.

Previous methods of fault determination after signature comparison have also not been documented. Two possible methods were tried. The first method used computer selection for each network function and manual correlation for fault isolation. The second method was completely programmed for digital computation.

As far as can be determined from available reports no investigator approached the problem of error due to in-tolerance variations of components. This study included an investigation of that problem with both the conventional and the proposed procedures.

II. PROCEDURAL BACKGROUND

A. FUNDAMENTAL CONCEPT

All components of a given network play a part in the formation of any of the network functions. The sensitivity of any given network function to a particular component is generally unique to that component. That is, in a network with K components a network function will have K sensitivity functions, one for each component. A change in the value of a particular component may cause a decrease in the magnitude of a network function at one frequency and an increase in the magnitude at a different frequency. There may in fact be frequencies at which the function sensitivity is near or equal to zero for a particular component, excluding catastrophic failures.

Specific knowledge about this behavior can be used to detect and isolate component value changes. If, for example, it is known that the input impedance is insensitive at low frequencies to changes in a specific component, one would assume that component was not faulty if the input impedance exhibits a significant change at low test frequencies.

B. POLE-SHIFT TECHNIQUE

The component variation effect on frequency response can be observed with the pole-shift technique. A network function can be represented by a constant and a unique set of poles and zeros; i.e.,

$$F(s) = K \frac{(s-Z_1)(s-Z_2) \cdots (s-Z_m)}{(s-p_1)(s-p_2) \cdots (s-p_n)}$$

where

$F(s)$ = any network function

K = constant

z_i = i^{th} zero frequency

p_j = j^{th} pole frequency.

Every component plays a part in pole and zero locations. However, a few components may dominate the location of a given pole or zero frequency. Thus, if one could detect the shift of the poles and zeros of a given function, it may be possible to determine component variations. A simple example can best illustrate this point.

Example 1 - Consider the network shown in Figure 1.

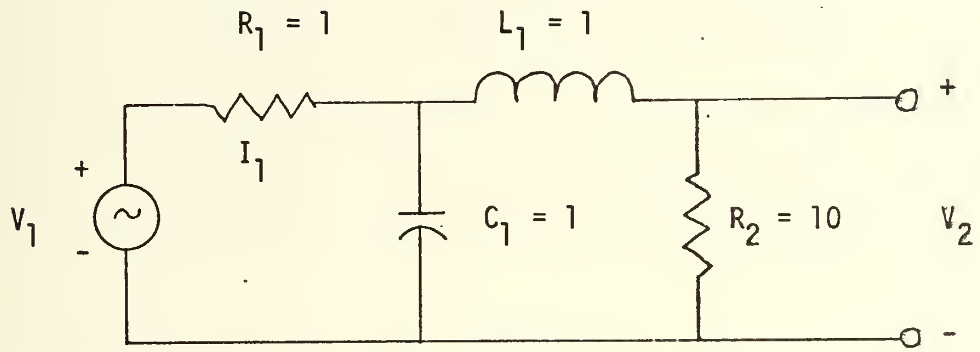

Figure 1. Example 1 - Low Pass Filter

The network function $\frac{V_1}{I_1}(s)$ is given by:

$$\frac{V_1}{I_1} = \frac{s^2 + 11s + 11}{s^2 + 10s + 1}$$

$$= \frac{(s + 9.887)(s + 1.113)}{(s + 9.899)(s + 0.101)}.$$

The pole and zero shifts were determined by varying each component value by ten percent and observing the corresponding critical frequency shifts. The results:

Circuit Condition	Location (radians/sec) p_1	p_2	z_1	z_2	Percent Shift p_1	p_2	z_1	z_2
Nominal	.101	9.899	1.113	9.887	0	0	0	0
R_1 variation	.101	9.899	1.020	9.889	0	0	.62	.02
C_1 variation	.092	9.908	1.010	9.899	8.9	.09	9.25	.12
L_1 variation	.101	8.990	1.114	8.977	0	9.2	.08	9.2
R_2 variation	.092	10.908	1.101	10.899	8.9	10.2	1.07	10.2

TABLE II-1. Percent Pole and Zero Shifts

It is immediately obvious that a change in R_2 or C_1 will cause a greater shift in z_1 than will a corresponding change in L_1 or R_1. Also, p_2 and z_2 are most affected by L_1 and R_2 and p_1 is affected only by C_1 and R_2.

C. CHANGES IN RESPONSE FROM POLE SHIFT

In practice the exact location of the critical frequencies can usually not be determined directly from the circuit. However, this does not rule out the pole shift technique. Any shift in critical frequency causes a change in the magnitude of the given function at all frequencies greater than the shifted one. This can be seen from the simple Bode representation of a pole:

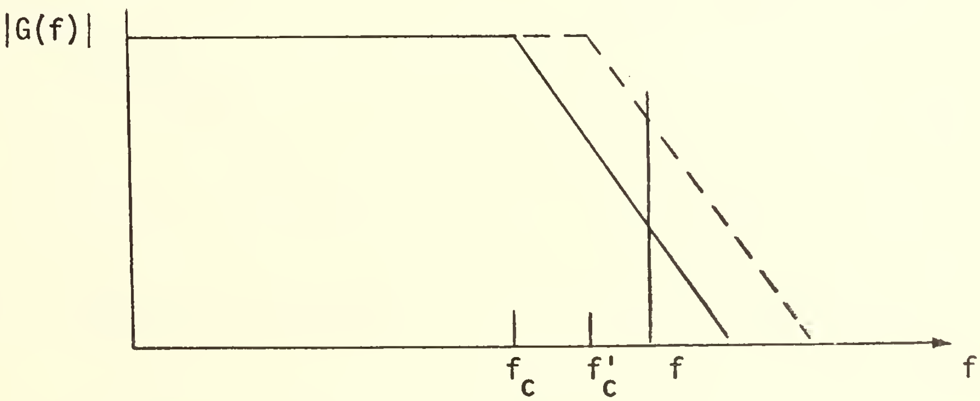

Figure 2. Bode Plot of Simple Pole

If f_c is changed to f_c' (indicated by dashed line), the function magnitude, at any frequency, f_o, greater than f_c, is changed (increased in this case). The magnitude is not affected at frequencies less than f_c.

Now, from the network in Figure 1 we obtain:

$$\frac{V_2}{V_1} = \frac{10}{s^2 + 11s + 11} = \frac{10}{(s + 9.887)(s + 1.113)}$$

Figure 3a is the Bode plot for the function V_2/V_1. Figure 3b and 3c show the effect of increasing R_1 and of increasing L_1, respectively. The nominal response is shown as a dashed line.

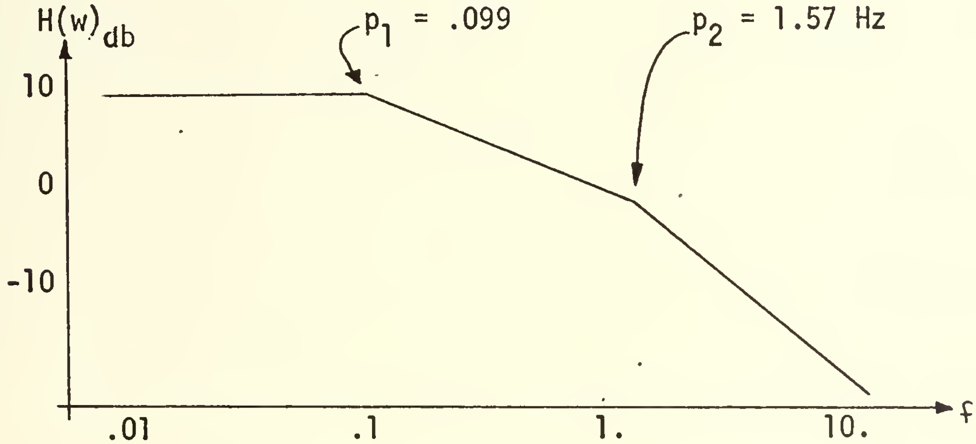

Figure 3a. V_2/V_1 Bode Plot.

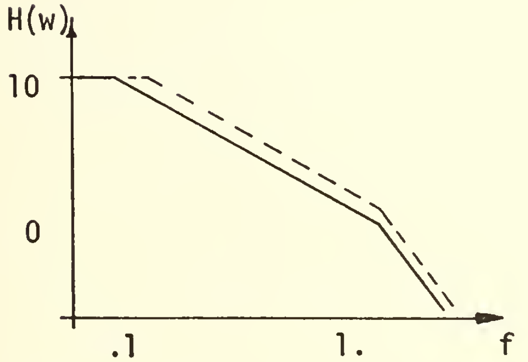

Figure 3b. V_2/V_1, R_1 Increased.

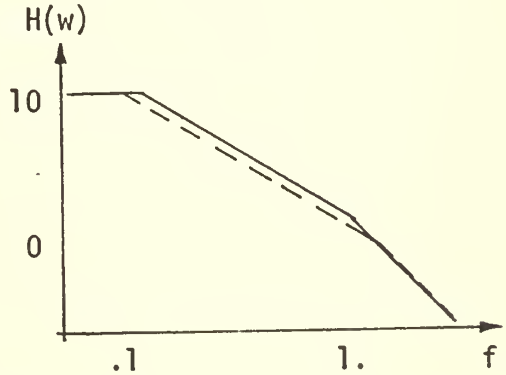

Figure 3c. V_2/V_1, L_1 Increased.

It can be seen that an increase in R_1 causes a decrease in the magnitude of the response between p_1 and p_2. A lesser decrease is seen at frequencies greater than p_2. The response between p_1 and p_2 is increased for an increase in L_1 while the response above p_2 is decreased. Thus, if one examines the response at a frequency between p_1 and p_2 and at a frequency greater than p_2 it will be a reasonably simple task to distinguish between an increase in R_1 or L_1. This example is an oversimplification of the procedure, but the application of which can be appreciated.

III. GENERAL PROCEDURE

A. BASIC PROCEDURE

The basic procedures outlined by Seshu and Waxman [1] was followed with modifications as explained herein. Two network functions were used concurrently. Work done by Maenpaa, Stehman, and Stahl [5] indicated that the redundancy from using two functions was more than adequately compensated for by the increased fault detection capability.

The symbolic network functions were calculated by hand. It should be pointed out that computer programs exist which calculate symbolic network functions. These programs should be used for more complex circuits.

The functions were numerically evaluated and factored to determine the poles and zeros. Test frequencies were then chosen. Previous procedures generally selected test frequencies based on the pole and zero locations. As a test of the procedure several test frequencies were chosen between the critical frequencies as well as 0 hertz and one frequency above the highest critical frequency. The actual selection was programmed for computer selection and is described in Section VII.

It was felt that the procedure of selecting test frequencies based on poles and zeros was, although intuitively satisfying, a somewhat illogical procedure. The concern was not with the function itself but with the behavior of that function with a parametric change. Thus, sensitivity functions were calculated for each element. Based on these functions, six test frequencies were chosen. This procedure is discussed in Section IV.

After selection of test frequencies, a worst case response, for component values within 10 percent tolerance limits, was computed for

each test frequency. This defined a nominal range of performance. This is not necessarily the best method for definition of nominal range. This range might be specified for a particular circuit. However, in the absence of such specifications the above procedure was followed.

Each component was then allowed to take on several discrete values out of tolerance limits. All other components were held fixed at their nominal value. The response at each test frequency was then calculated and catalogued for the entire collection of sets of component values.

The catalogued response values were then quantized into nine levels and labeled one through nine. Quantization level five was reserved for response within nominal limits. The setting of the levels is explained in Part B. The quantized responses for a given set of component values is defined as the signature for the given circuit configuration.

The signatures were computed for the following cases:

value = zero
$\pm$ 20%
$\pm$ 50%
100 times nominal.

The entire set of signatures for a given function is defined as the library for that function. There is no theoretical limit on the size of the library. However, the inclusion of many more circuit configurations would probably only complicate the matching procedure and not increase detection capability.

Using the primary test circuit the input impedance and voltage gain at each test frequency was measured. Then, after quantization, the signatures measured were compared with the library signatures and a "closest match" was made. Thus, the faulty component was identified.

In order to test the procedure, several error conditions were set and the response calculated. In order to more closely approximate actual

circuit conditions, all component values were allowed to vary randomly (uniform distribution) within 10 percent tolerance limits. Calculations were made with and without the random variations to determine how much effect this would have on the results.

The results of all tests are presented in Section VI.

B. SETTING OF QUANTIZATION LEVELS

With the exception of selection of test frequencies the most critical step in the procedure was the setting of quantization levels. Two methods were used successfully. One was programmed as part of the main analysis program. The second method, although not programmed is possibly adaptable to computer selection. Both methods have nine levels of response with level five reserved for the nominal range.

The nominal range was selected by taking the range between the maximum positive and maximum negative deviation from normal for any element value at $\pm$ 10% from normal. The programmed method then set the other eight levels as shown in Table III-1 in which F is a multiplying factor used to force the possible response range to cover all quantum levels with levels one and nine assigned only for extreme cases.

Quantum Level	Normalized Response
1	$(\infty, 1 + 13\ FN]$
2	$(1 + 13\ FN, 1 + 7\ FN]$
3	$(1 + 7\ FN, 1 + 4\ FN]$
4	$(1 + 4\ FN, 1 + N]$
5	$(1 + N, 1 - M)$
6	$[1 - M, 1 - 4\ FM)$
7	$[1 - 4\ FM, 1 - 7\ FM)$
8	$[1 - 7\ FM, 1 - 13\ FM)$
9	$[1 - 13\ FM, 0)$

TABLE III-1. Quantization levels.

The second method used a more inituitive approach. The maximum deviation levels, 1 and 9, were selected based on the maximum deviation from normal under any condition. The mid levels were then selected so that the catalogued or library signatures would cover all quantum levels as uniformly as possible.

The manual approach is acceptable for small circuits but gets unmanageable quickly as circuit size increases. Parts of the procedure could be easily programmed. However, a suitable algorithm would have to be developed prior to complete programming.

C. TWO COMPONENT VARIATIONS

It cannot be assumed that only one component at a time will undergo changes in value. In order to investigate the possibility of two component values changing simultaneously, two possibilities were considered. If one component masks function changes due to variations in another component (i.e., no change in function magnitude), the entire procedure will not work. However, if one is primarily concerned with circuit performance, the failure to detect component variations is of no concern if there is no degradation in circuit performance.

The second situation considered was one in which there was an obvious change in circuit response due to two components varying. The original procedure made allowances for varying only one component at a time. However, a slight modification allowed for the generation of a set of library signatures for two component variations. The set is presented in the computer output section.

It soon became apparent that in order to use the two component variation library, one needed apriori knowledge of the number of faulty components. Since this is not usually available, the usefulness of two

component signatures is doubtful. Additionally, an examination of the library signatures revealed that one of the two components varied usually dominates the response. Thus, if two components have failed the single component library would probably isolate the dominant component. After replacement of that component further testing could reveal the second faulty component. Consequently no further investigation of multi-component variation was undertaken.

IV. PRIMARY TEST CIRCUIT

The primary circuit used throughout the investigation was a six-element low-pass filter. (See Figure 4.) The unscaled frequency response, represented by the voltage ratio transfer function, V_2/V_1, is shown in Figure 5 and the unscaled input impedance as a function of frequency is shown in Figure 6. On Figures 5 and 6 the poles are represented by X and the zeros by O.

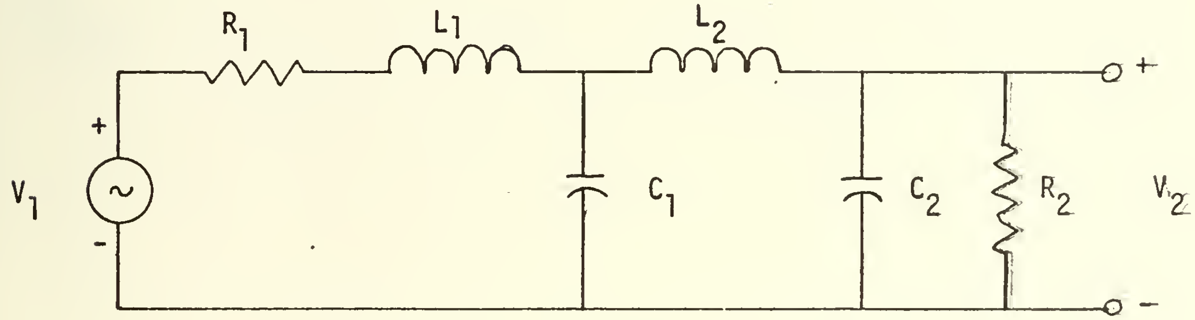

Figure 4. Primary Test Circuit.

The component values were magnitude scaled by 100 and frequency scaled by 10^5. This yielded component values of approximately the same magnitude. Ideally, the scaled cutoff frequency should be 1 Hz. However, with this scaling f_c = .02 Hz and no difficulty was encountered.

The scaled network functions of interest are:

$$\frac{V_2}{V_1} = \frac{.1634}{s^4+17.67s^3+6.462s^2+2.601s+.2124}$$

$$= \frac{.1634}{(s+17.31)(s+.0997)(s+1.325+j3.25)(s+1.325-j3.25)}$$

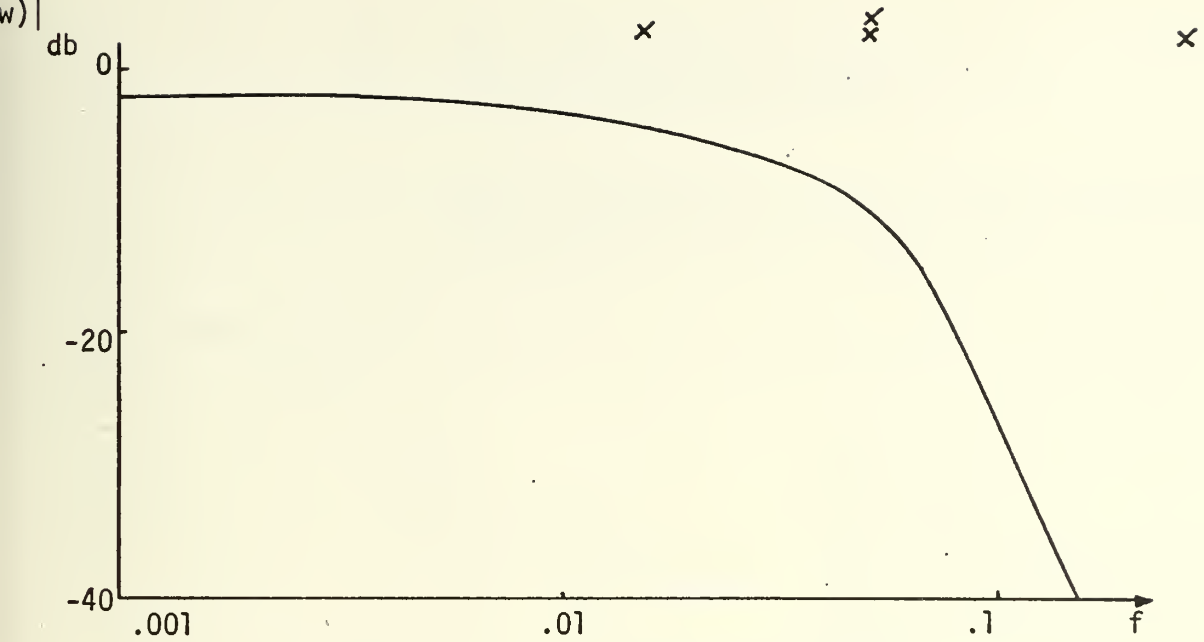

Figure 5. V_2/V_1, Primary Test Circuit

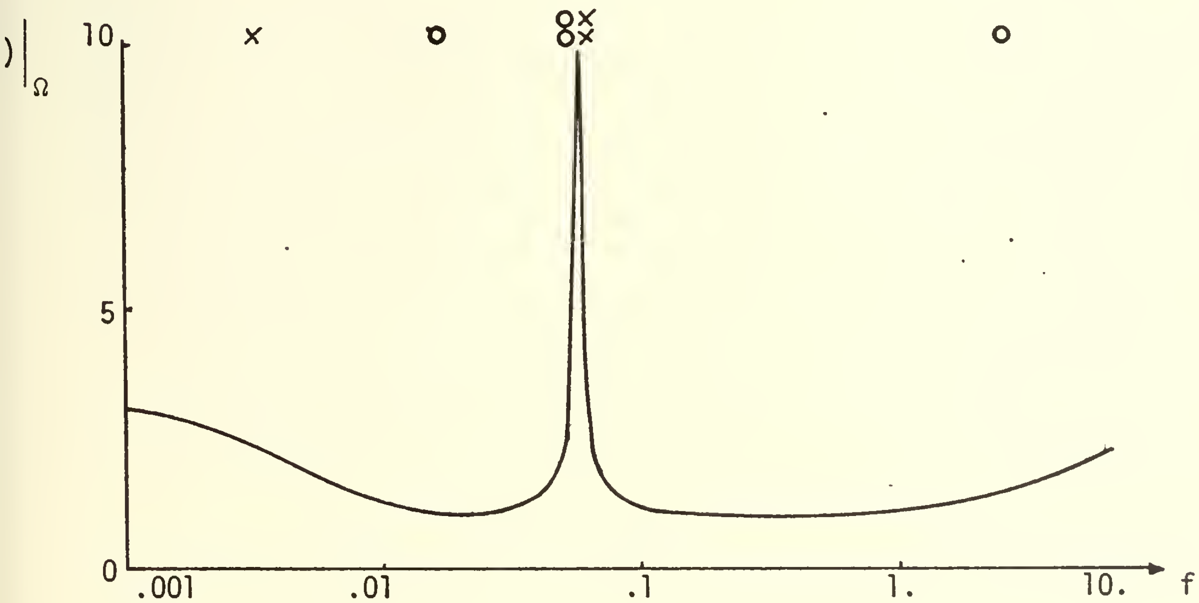

Figure 6. V_1/I_1, Primary Test Circuit

$$\frac{V_1}{I_1} = \frac{s^4+17.67s^3+6.462s^2+2.601s+.2124}{5.882s^3+.147s^2+.817s+.01634}$$

$$= \frac{(s+17.31)(s+.0997)(s+1.325+j3.25)(s+1.325-j3.25)}{(s+.02)(s+.0025+j.373)(s+.0025-j.373)}$$

The poles and zeros in Hz are listed in Table IV-1 for convenience.

V_1/I_1 Zeros* (Hz)	V_1/I_1 Poles (Hz)
.0518 (Second Order)	.0594 (Second Order)
.0159	.00319
2.76	

* Zeros of V_1/I_1 are the poles of V_2/V_1.

TABLE IV-1. Pole and Zero Magnitudes.

V. SELECTION OF TEST FREQUENCIES

A. CONVENTIONAL METHOD

The conventional procedure was followed with regard to selection of test frequencies in order to test the method. It soon became apparent that the number of test frequencies was too large to handle with ease. Thus, only frequencies between the pole and zero frequencies were selected as well as 0 Hertz and one above the maximum critical frequency.

The computer program written for implementation of the procedure (see Program 1) did the frequency selection automatically. The test frequencies were selected by taking

$$f_{test} = f_c^i \pm \frac{f_c^{i+1} - f_c^i}{2}$$

for every f_c^i where $f_c^i = i^{th}$ critical frequency. If

$$\left[f_c^i + \frac{f_c^{i+1} - f_c^i}{2} \right] > \left[f_c^{i+1} - \frac{f_c^{i+2} - f_c^{i+1}}{2} \right]$$

one of the frequencies was eliminated. Test frequencies were computed for the input impedance because the critical frequencies for V_2/V_1 are included in the critical frequencies for V_1/I_1. One test frequency was eliminated due to the above inequality. This resulted in the selection of the following seven test frequencies for a fourth order filter:

0.0	Radians
.001	Radians
.00498	Radians
.196	Radians
.5488	Radians
8.49	Radians
25.8	Radians

TABLE V-1. Conventional Method Test Frequencies

B. SENSITIVITY APPROACH TO FREQUENCY SELECTION

Inspection of the signatures generated using the conventional method showed that redundancy existed to the extent that the method was somewhat inefficient. Two test frequencies, D.C. and the lowest nonzero test frequency, gave nearly identical results. This indicated one of the two frequencies was probably not required. It was also felt that unless further investigation was undertaken it would be possible to ignore a range of frequencies which could be extremely helpful.

This investigation took the form of calculating and plotting* functions which defined the sensitivity of V_2/V_1 and I_1/V_1 to variations in each component. These plots are presented as Figures 7 through 18. Figure 19 is a superposition of the plots of the sensitivity function for I_1/V_1 and Figure 20 is the same for V_2/V_1. Ideally V_1/I_1 should have been used. However, the computer program used in this portion of the study did not allow for using V_1/I_1. It was felt that no loss of information would result.

The superposition plots Figures 19 and 20 are very revealing. The first observation is that V_2/V_1 is not sensitive to changes in R_1 and R_2 at high frequencies nor to C's or L's at low frequencies. While this information is not new the sensitivity functions assist in determining when C's and L's have little effect. This can be used to eliminate the redundancy observed using the conventional method of frequency selection.

The mid-frequency range is the most interesting range for the selection of test frequencies. For example, near .02 Hz V_2/V_1 is very sensitive to changes in R_1 and C_2, less sensitive to changes in L_2, even less sensitive

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* The program NASAP-70 was used to calculate the various sensitivity functions.

to changes in R_2 and C_1 and virtually insensitive to changes in L_1. There is a peaking of the sensitivity functions for C_1, C_2, and L_2 near .06 Hz. At this same frequency the sensitivity functions for R_1, R_2, and L_1 are near zero.

The $I/_1$ and V_1 plots show that at near .3 Hz only the sensitivity function for R_1 is high. The functions for L_1 and C_1 are low and the other functions are zero. At 3 Hz I_1/V_1 is about equally sensitive to changes in L_1 and R_1 while the other functions are very close to zero. It can also be seen that the frequencies discussed in connection with V_2/V_1 are reasonably good choices for I_1/V_1.

Based on the above observations the following six frequencies were selected as test frequencies:

Number	Frequency
1	.00048 Hz
2	.027 Hz
3	.06 Hz
4	.299 Hz
5	2.99 Hz
6	49.97 Hz

TABLE V-2. Sensitivity Function Test Frequencies.

After some preliminary work it was observed that the response at .06 Hz was not well behaved. Since four sensitivity functions peaked for I_1/V_1, random variation within tolerance limits of these components were being observed as significant changes in response. In order to overcome this problem, test frequency three was changed to .055 Hz. The new frequency was off the peak enough to be useable and still yield the desired information.

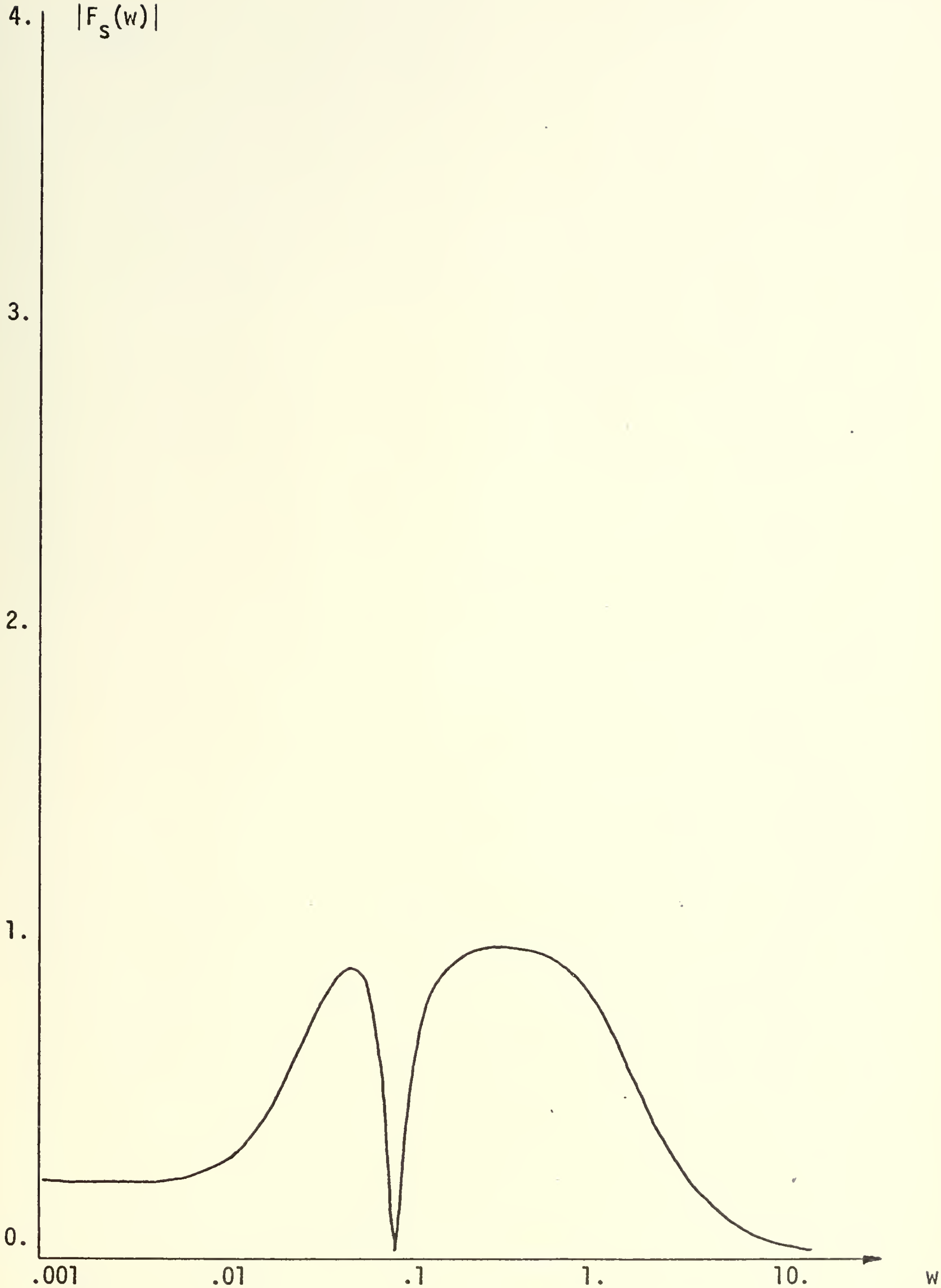

Figure 7. Sensitivity of V_2/V_1 to R1.

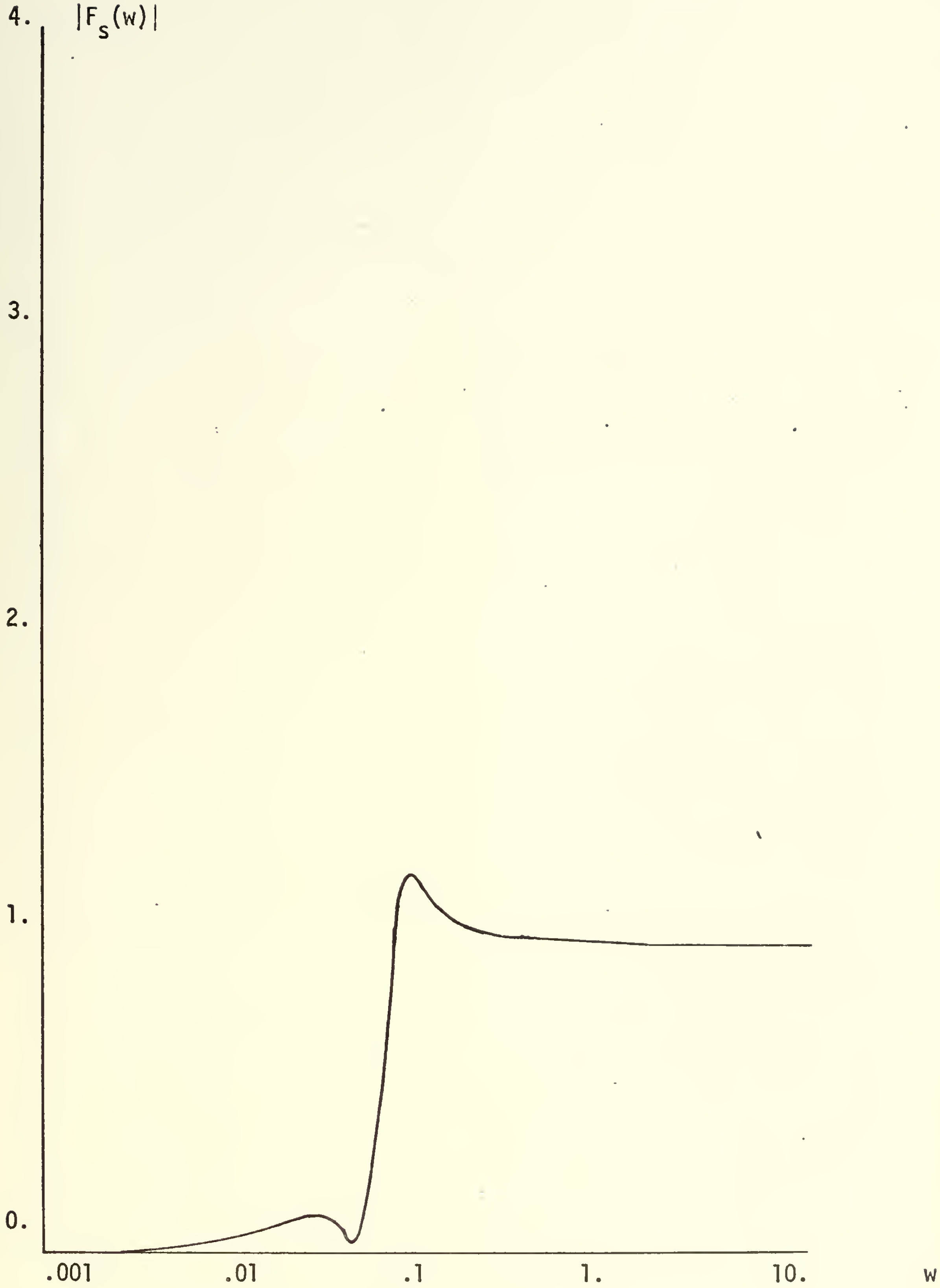

Figure 8. Sensitivity of V_2/V_1 to L1.

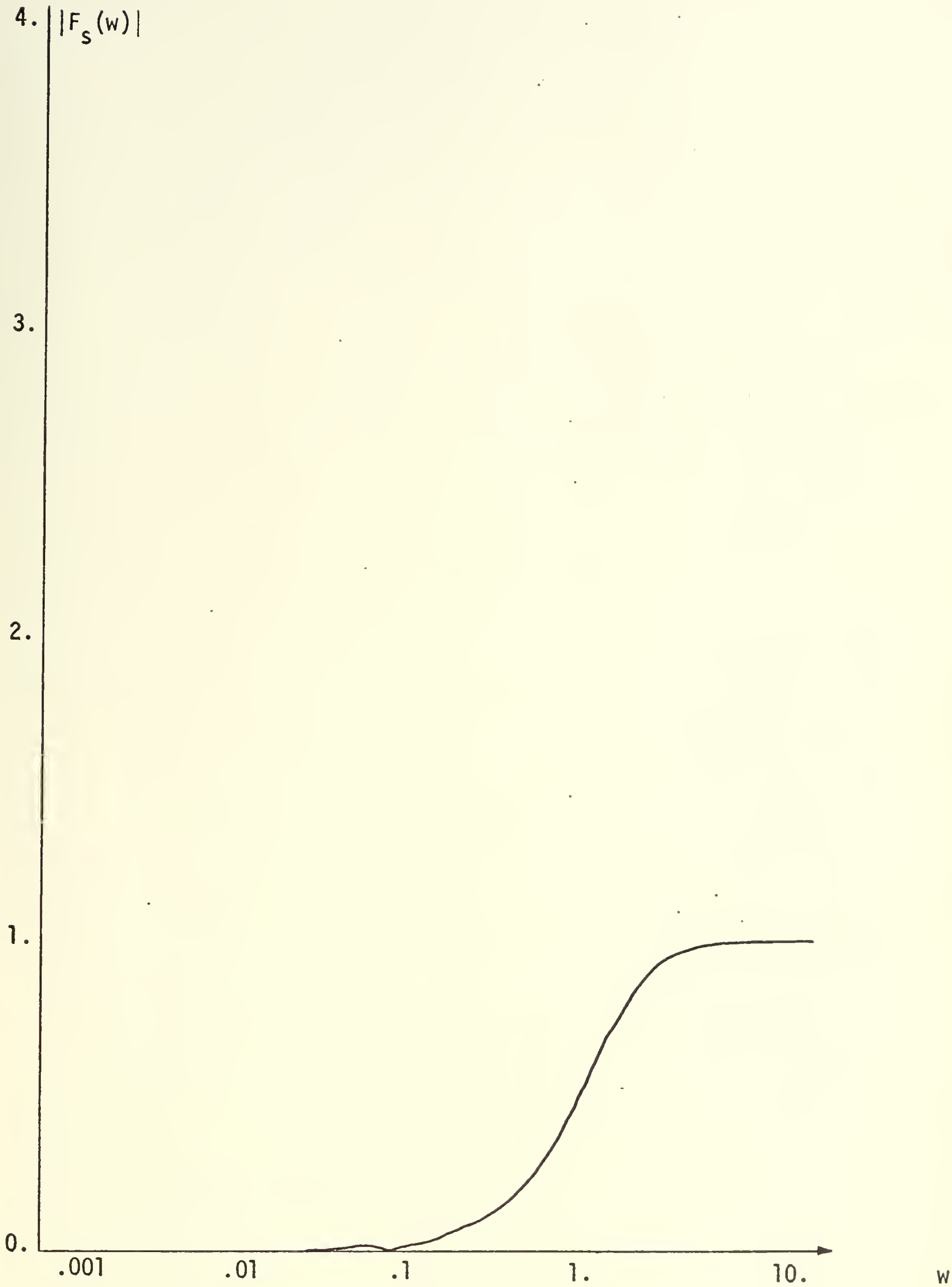

Figure 9. Sensitivity of V_2/V_1 to C1.

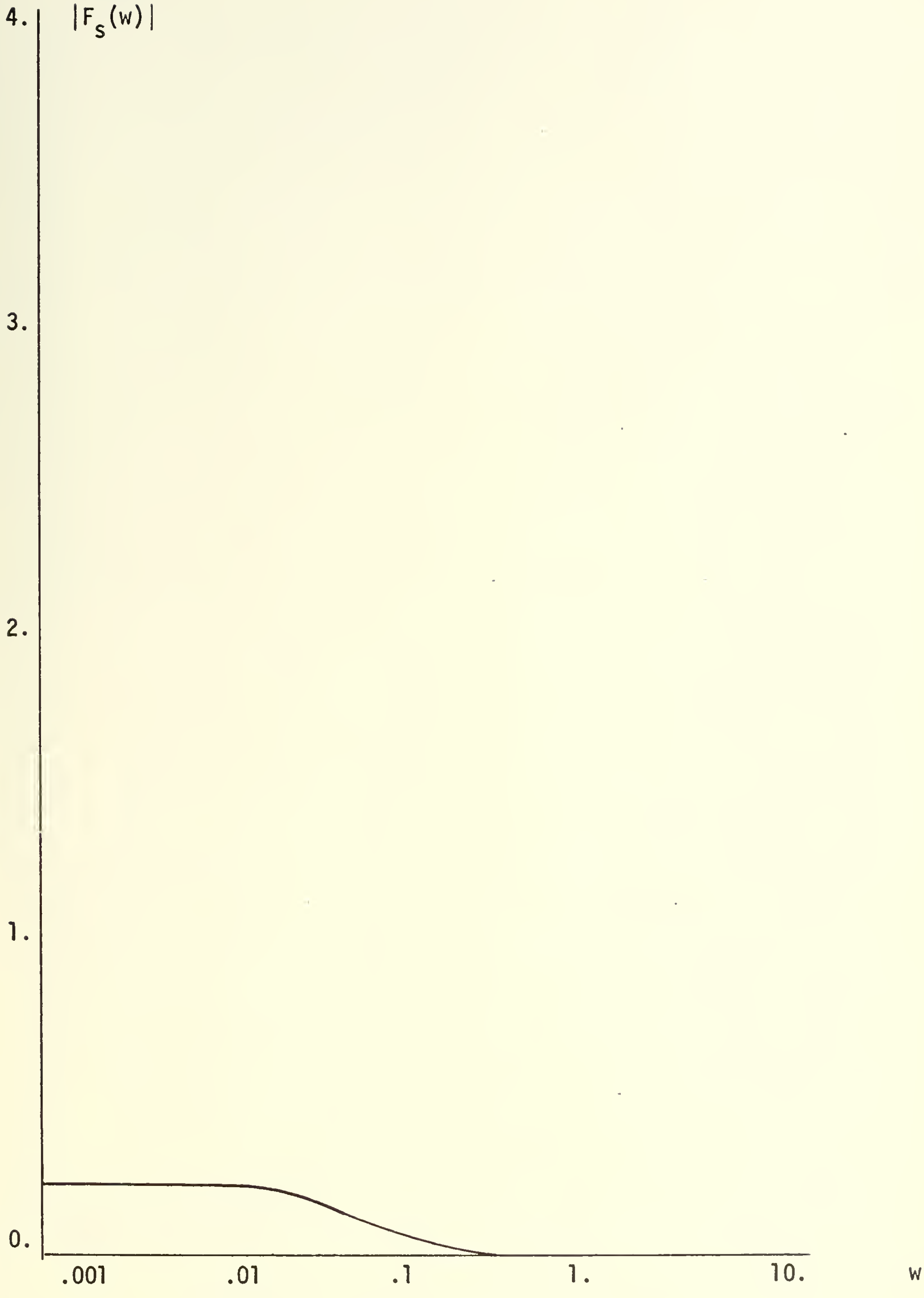

Figure 10. Sensitivity of V_2/V_1 to R2.

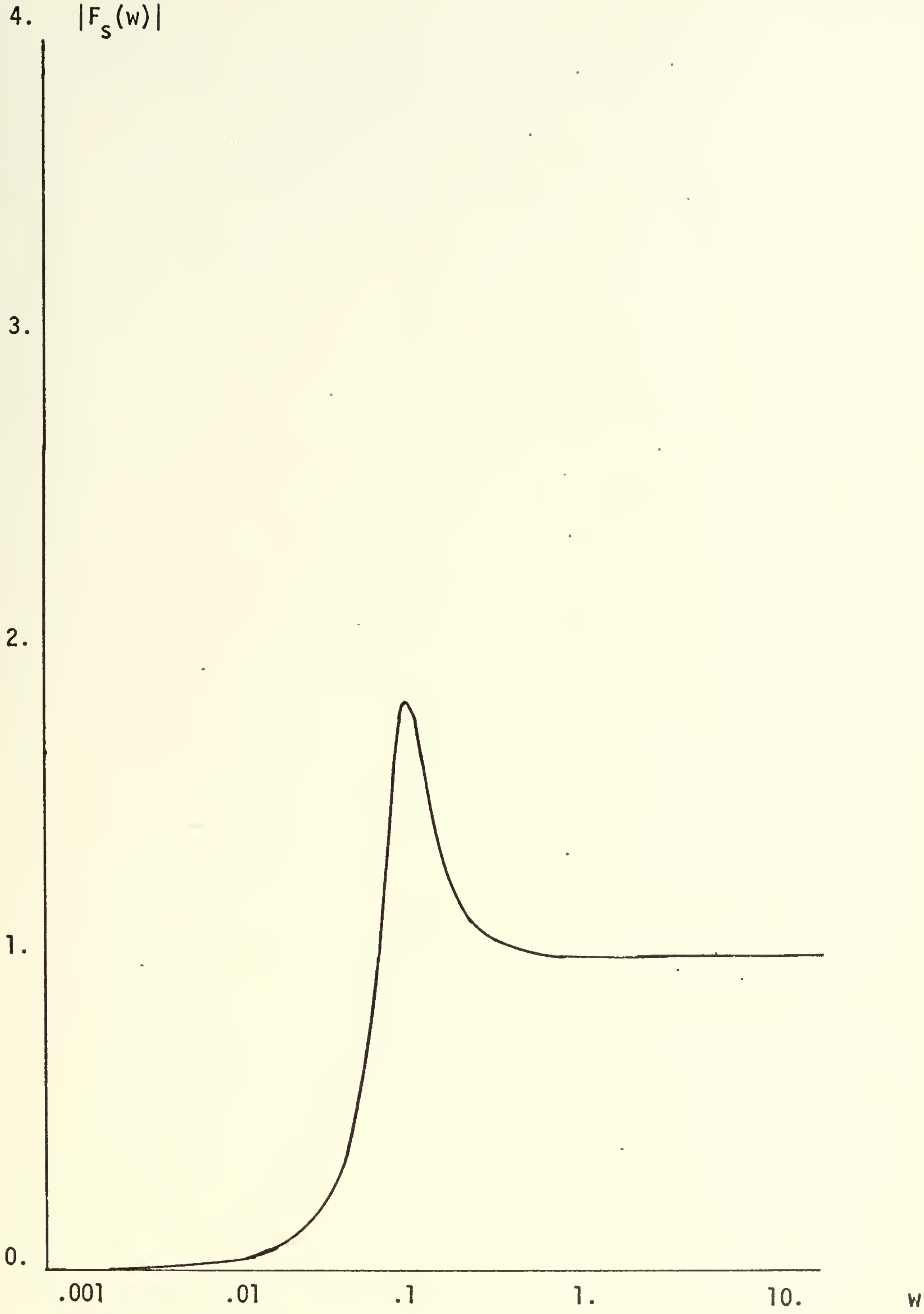

Figure 11. Sensitivity of V_2/V_1 to L2

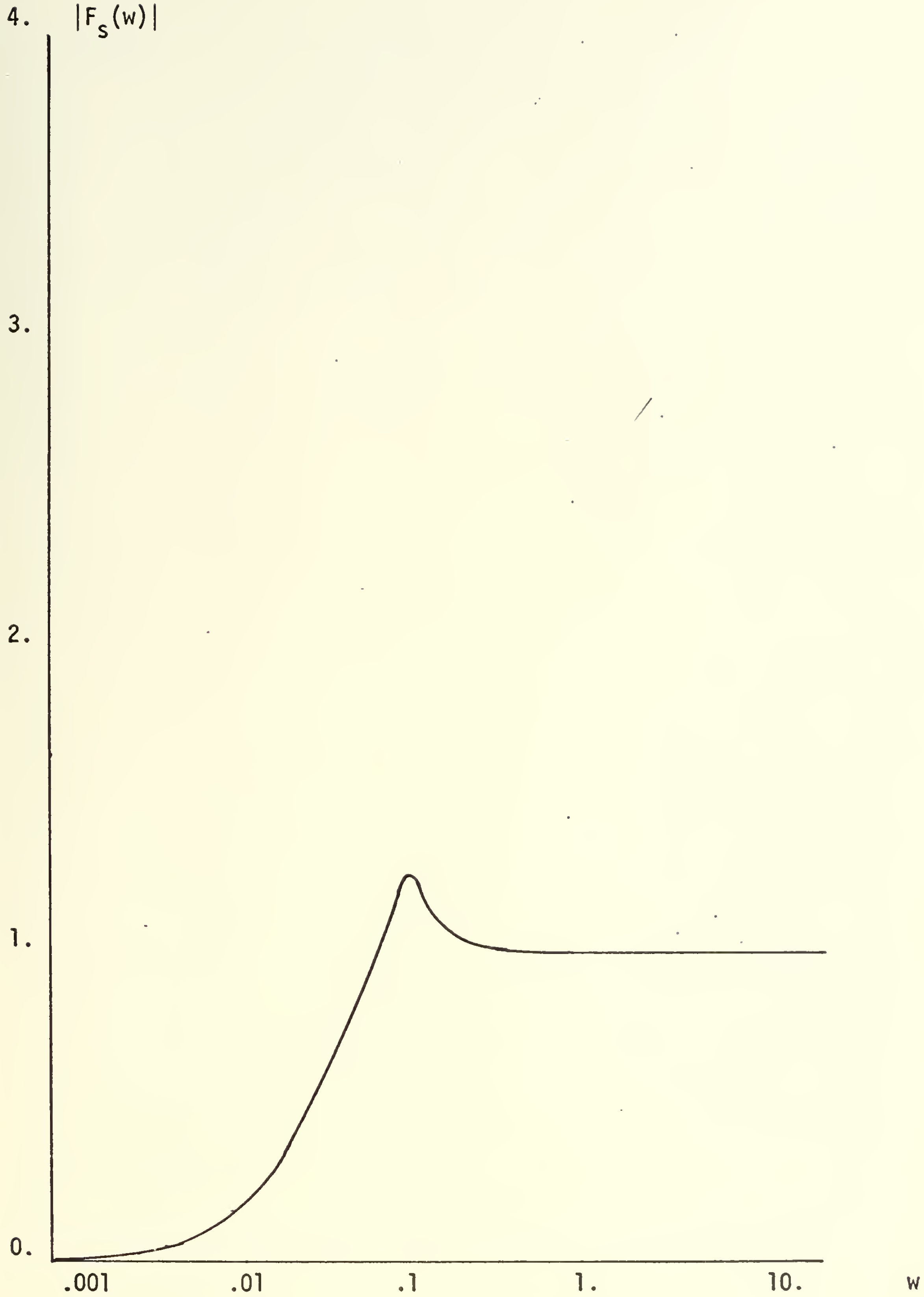

Figure 12. Sensitivity of V_2/V_1 to C2.

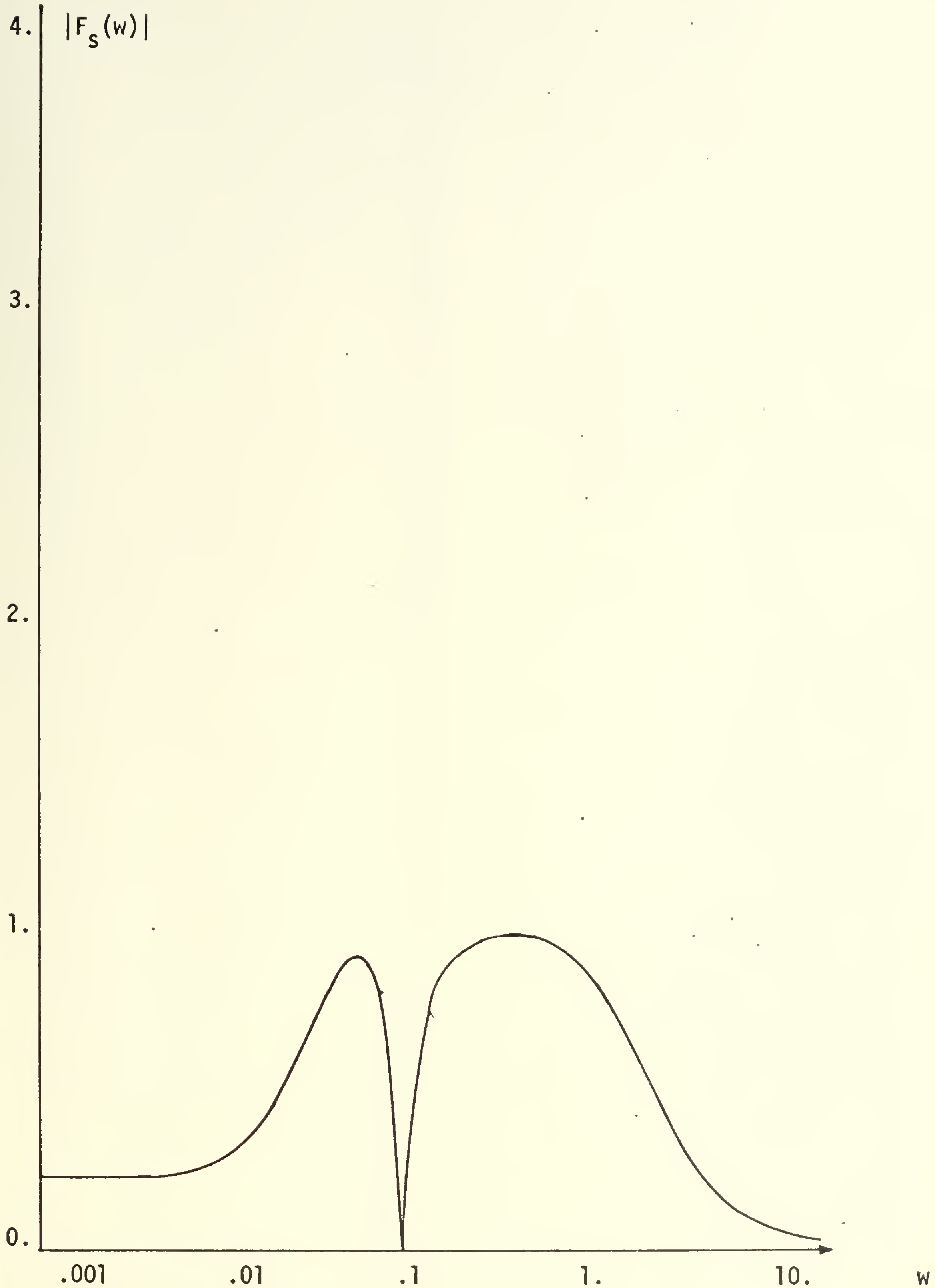

Figure 13. Sensitivity of I_1/V_1 to R1.

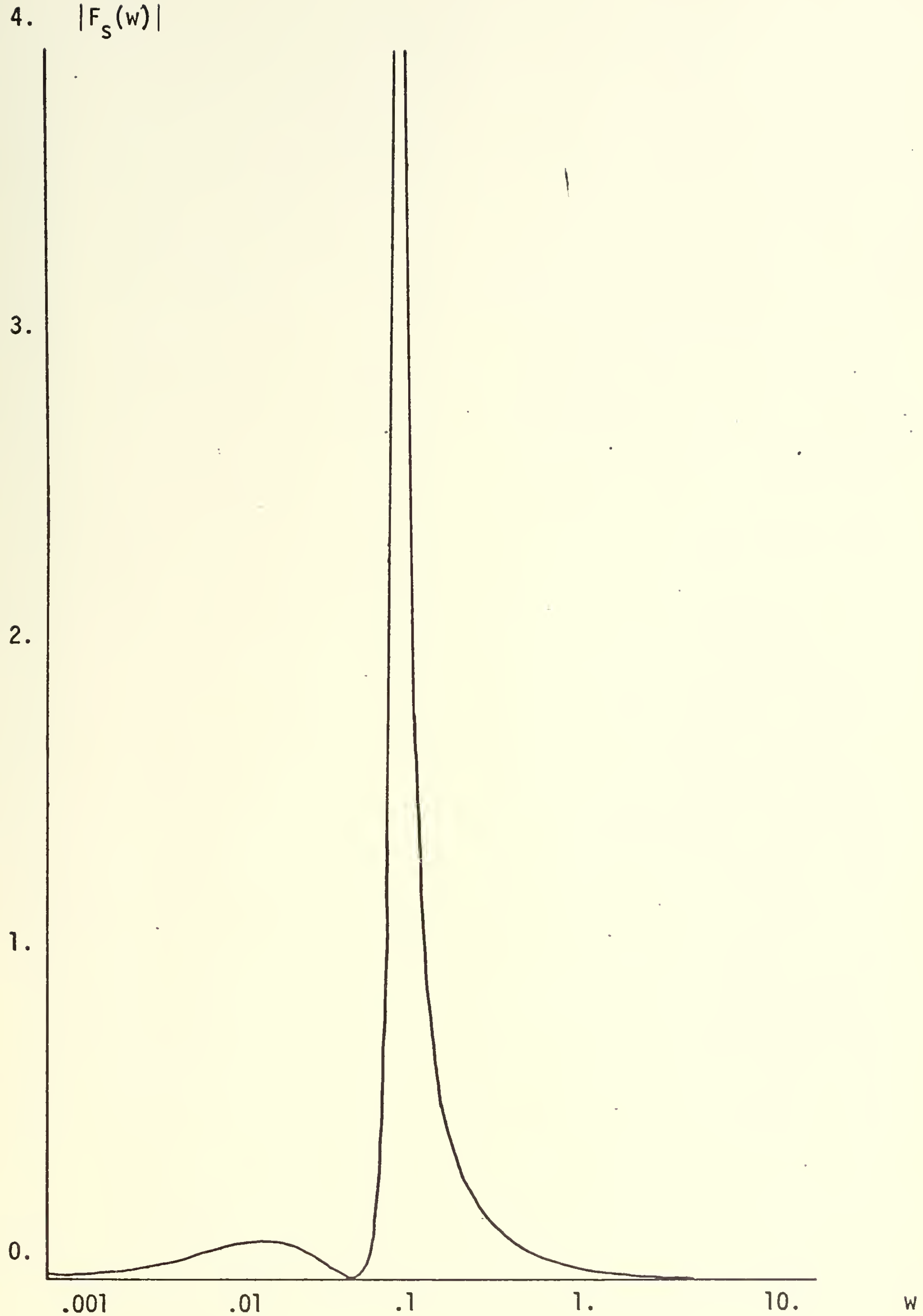

Figure 14. Sensitivity of I_1/V_1 to L1.

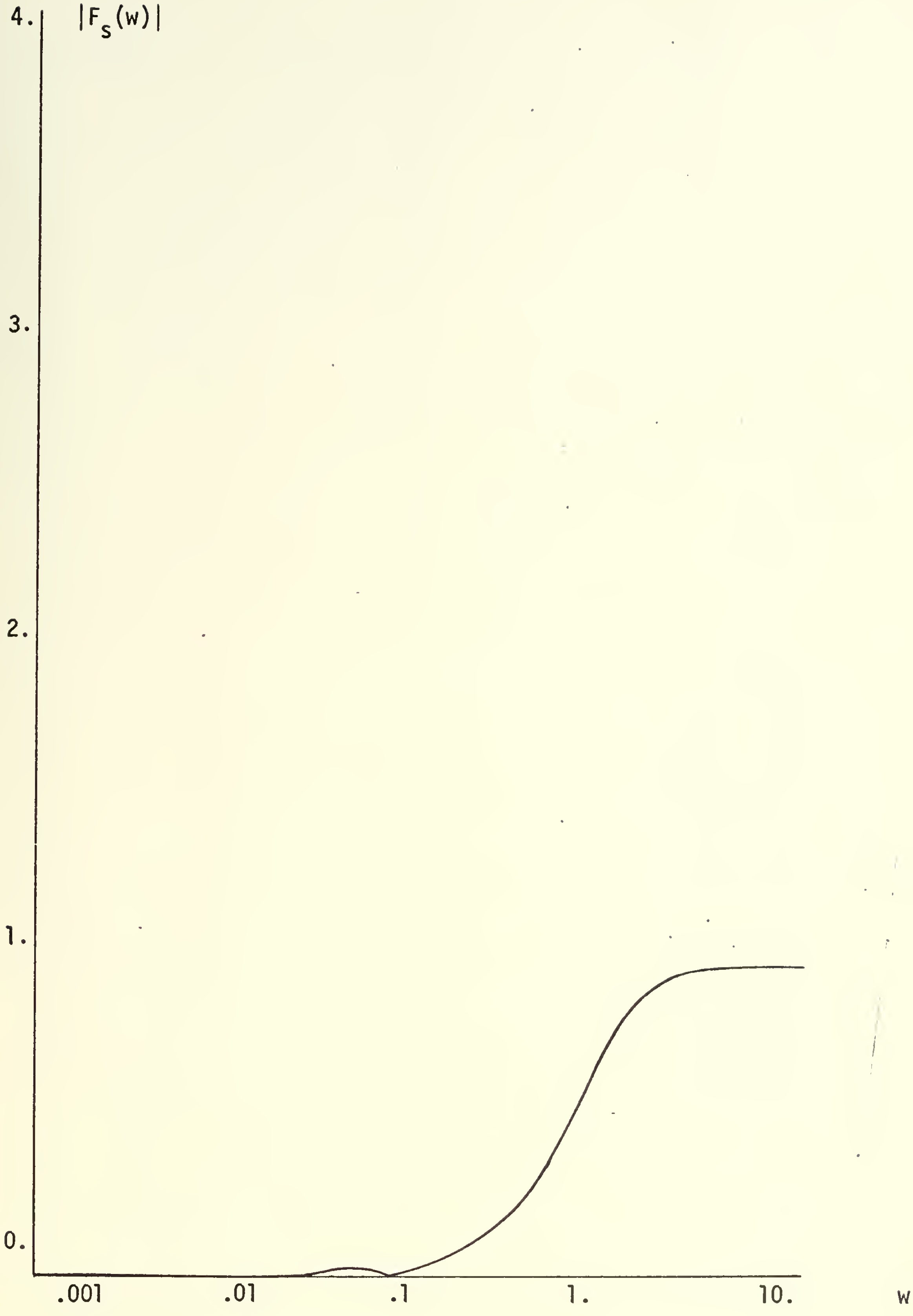

Figure 15. Sensitivity of I_1/V_1 to C1.

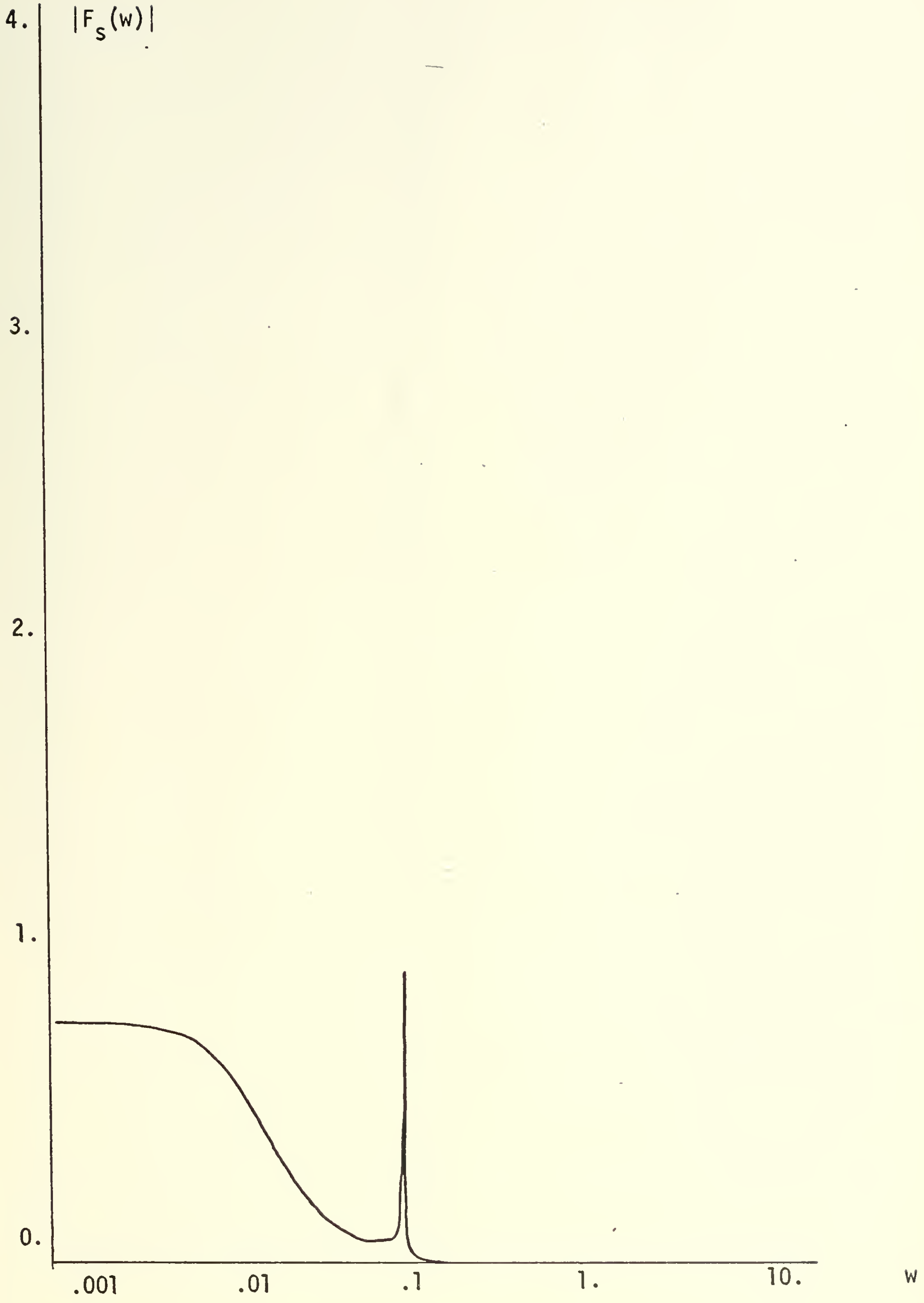

Figure 16. Sensitivity of I_1/V_1 to R_2.

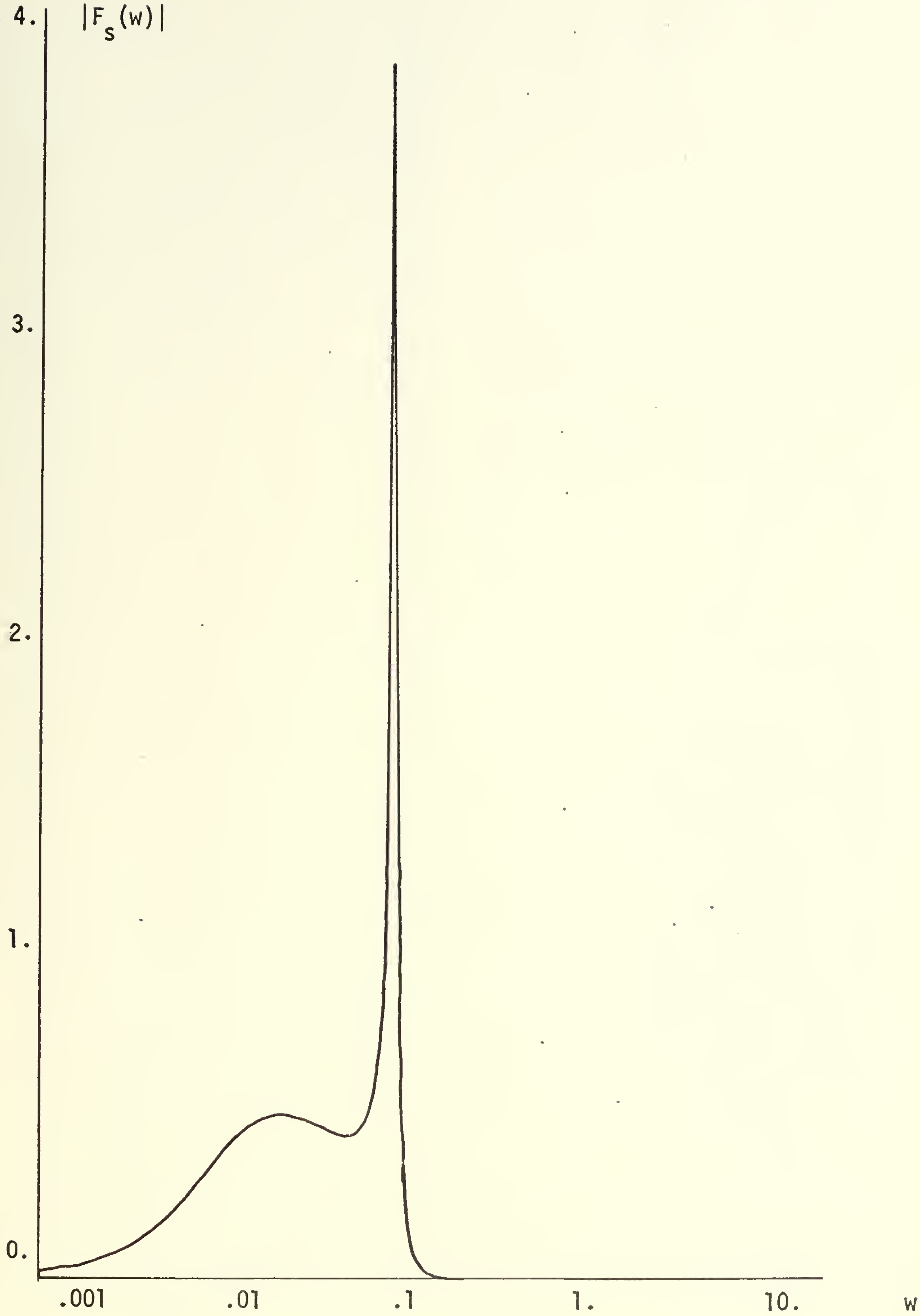

Figure 17. Sensitivity of I_1/V_1 to L2.

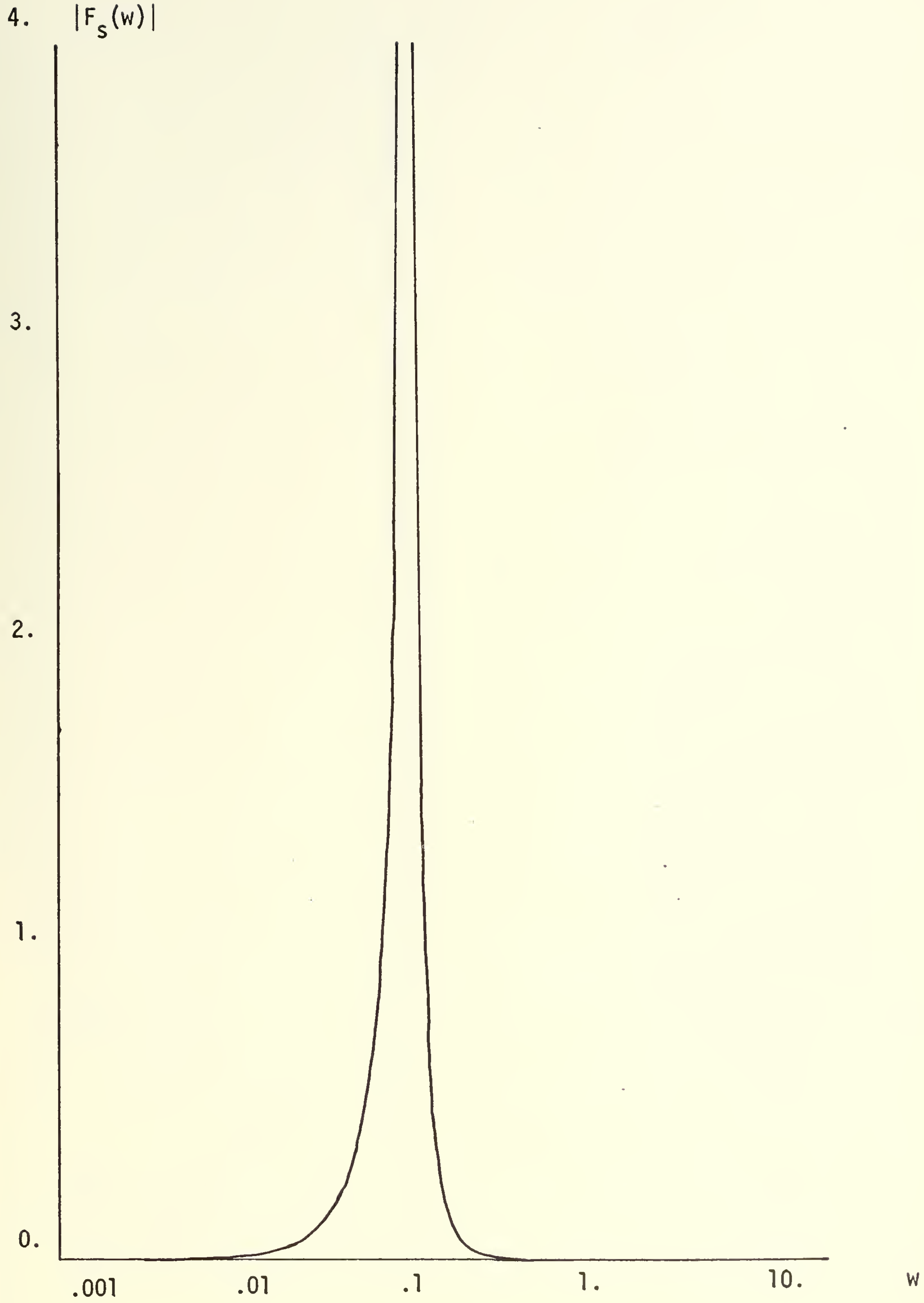

Figure 18. Sensitivity of I_1/V_1 to C2.

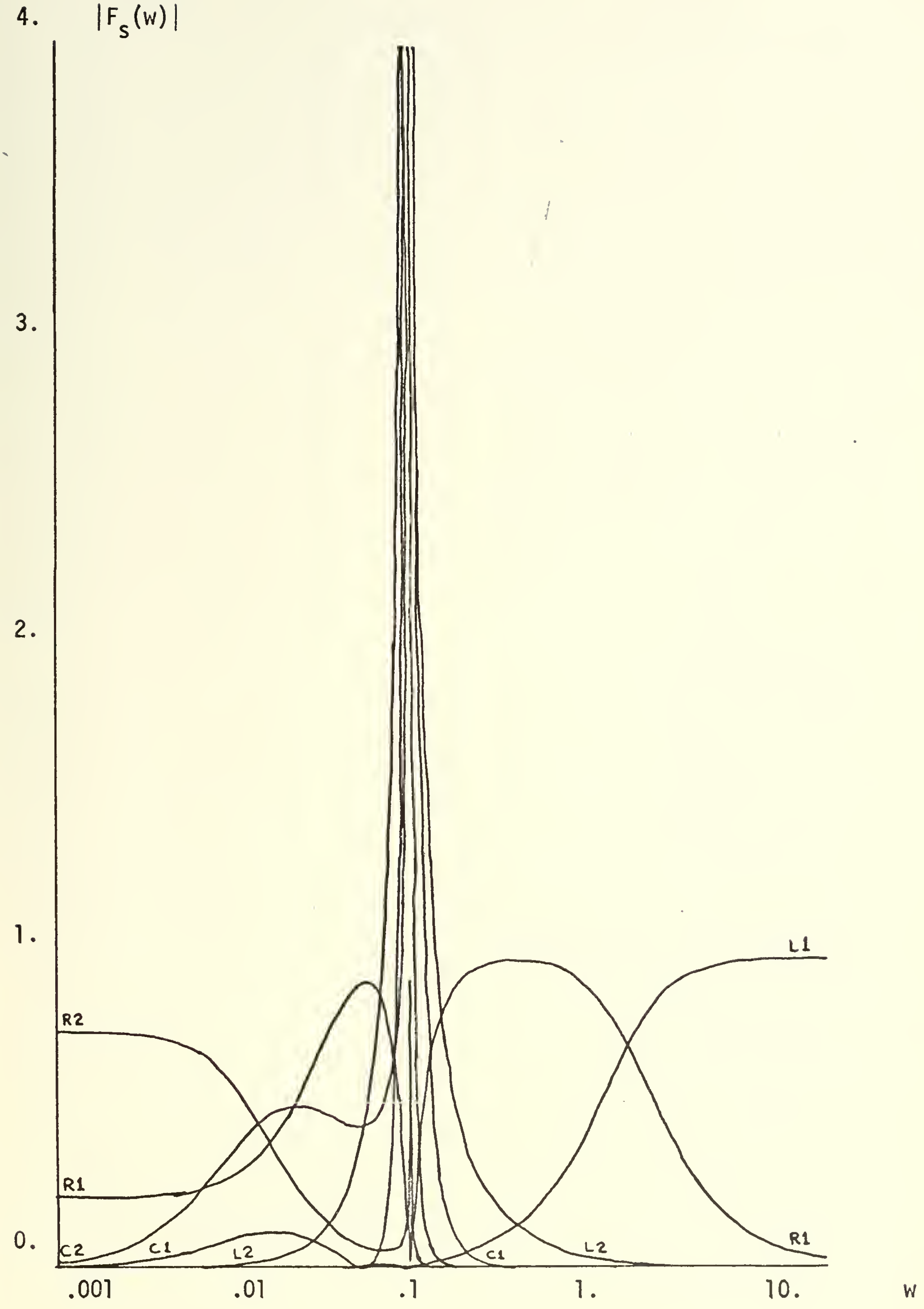

Figure 19. I_1/V_1 Composite Sensitivity Functions.

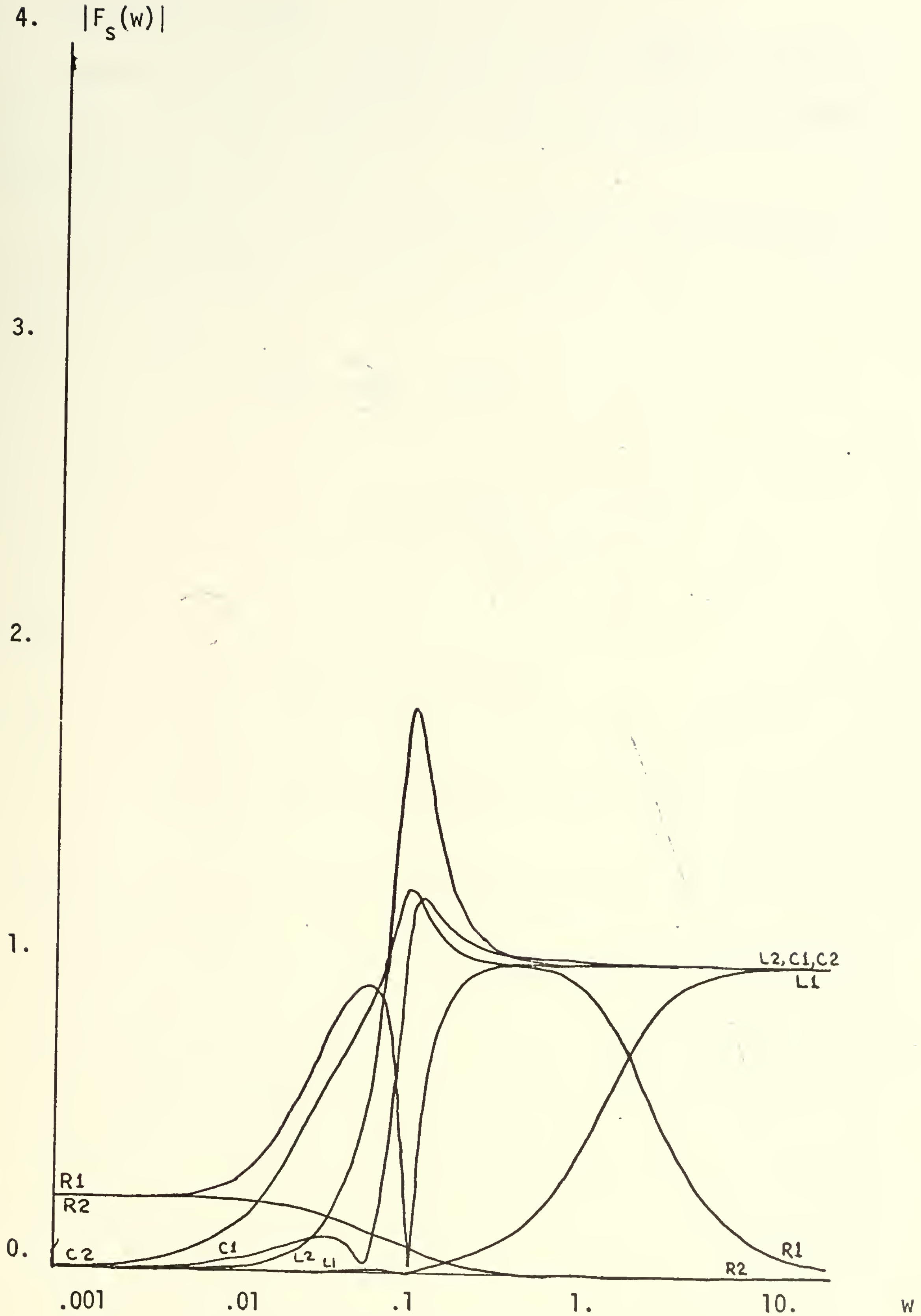

Figure 20. V_2/V_1 Composite Sensitivity Functions.

C. COMPARISON OF METHODS

The comparison of the results using the two methods of frequency selection is included in Section VI. Some comments as to applicability should be made at this point.

If one computes sensitivity functions by using the derivative of the network function with respect to the element under consideration it becomes

$$f_s(s) = \frac{d}{de_k} \frac{p(s)}{q(s)}$$

$$= \frac{q(s)\ \frac{dp(s)}{de_k} - p(s)\ \frac{dq(s)}{de_k}}{[q(s)]^2}$$

where:

$f_s(s)$ = sensitivity function

d/de_k = derivative with respect to element k

$\frac{p(s)}{q(s)}$ = ratio of polynomials representing the network function.

Thus, the poles of the sensitivity function are the same as the poles of the original function. Consequently, if one selects test frequencies on the basis of function poles the selection is linked to the sensitivity functions. Generally, however, the zeros of the sensitivity function are unique and are thus ignored by any selection procedure based on the poles and zeros of the original function.

From the preceding paragraph, some of the advantages of using sensitivity functions for test frequency selection become apparent. However, that method has the disadvantage of requiring the calculation of sensitivity functions for each element.

With the small circuit used as an example the value of sensitivity functions are demonstrated. With more complex circuits it appears more probable that important test frequencies might be overlooked if the sensitivity functions are not used.

VI. RESULTS WITH PRIMARY CIRCUIT

A. RESULTS USING BASIC PROCEDURE

Using the procedures outlined in Section III-A and V-A a set of signatures was generated for the six selected circuit conditions. These signatures are presented in Figure 25. A second set of signatures was then generated for the same circuit conditions with random variation allowed for the circuit elements. The second set of signatures was input to the selection program to determine what errors resulted from the random variations.

Table VI-1 lists the input condition and the program output. The numerical result was determined as follows:

1 = Selection was correct condition
2 = Selection was correct element
3 = One of multiple selections was the correct condition
4 = One of multiple selections was correct element
5 = Incorrect selection.

By correlating the results from V_2/V_1 and V_1/I_1 a "hit", "miss", or "possible hit" rating was assigned for each test signature as follows. A hit was assigned if both V_2/V_1 and V_1/I_1 yielded a 1 or 2 or if one was 1 or 2 and the other was 3 or 4. A 3-5, 4-5, or 5-5 combination was called a miss. All other combinations were called possible hits. For some cases deviations were warranted. For example, a 3-4 selection could have been called a miss if a component other than the given component was clearly indicated as being faulty.

The result was 21 hits, 9 possible hits and 6 misses for 58% hits and 83% hits or possible hits. Since only the circuit conditions used for signature generation were used to generate the test signatures 100%

Circuit Condition	Program Condition	Numerical Result	Program Condition	Numerical Result
R1: Short	R1: Short	1	R1: Short	1
-50%	R1: -50%	1	L2: Open	5
-20%	R1: Open	2	L2: +50%	5
+20%	L2: +20%	5	R1: +20%	1
+50%	R1: +20%	2	R1: +20%	2
Open	R1: Open	1	L2: Short	
			R1: Open	3
L1: Short	L1: Short	1	L1: -50%	2
-50%	L1: -50%	1	L1: -50%	1
-20%	L1: -20%	3	L1: -20%	1
	R2: +20%			
+20%	L1: +20%	1	L1: -20%	2
+50%	L1: +20%	2	L1: -20%	2
Open	L1: Open	1	R2: +20%	5
C1: Open	C1: Open	1	L1: +50%	5
-50%	C1: +20%	4	C1: -50%	1
	C2: -50%			
-20%	C2: +20%	5	R1: -20%	5
+20%	L1: +20%	5	L1: -50%	5
	C2: +20%			
+50%	C2: +20%	5	C1: +20%	3
			C2: +20%	
Short	C1: Short	1	C1: Short	1
R2: Short	R2: -50%	2	R2: Short	1
-50%	R2: -50%	1	R2: -50%	2
			R2: -20%	
-20%	R2: -20%	3	R1: -20%	5
	L2: +20%			
	C2: +20%			
+20%	C2: +20%	5	C1: +20%	5
			C2: +20%	
+50%	R2: +20%	2	R2: +20%	2
Open	R2: Open	1	C1: Open	5

TABLE VI-1. Basic Procedure Results.

L2: Short	L2: Short	1	L2: Short R1: Open	3
-50%	L2: -50%	1	L2: -50%	1
-20%	L2: -50% L1: +20%	2	L2: -20%	1
			L2: +20%	1
+20%	L2: +20% C2: +20%	3		
+50%	L2: +20%	2	L2: Open	2
Open	L2: -20%	2	L2: Open	1
C2: Open	C1: +20% C2: -50%	4	R2: +20%	5
-50%	C1: -20%	5	C1: +50% C1: -20% C2: -20%	4
-20%	Nominal	5	L2: +20%	5
+20%	C2: +20%	1	C2: +20%	3
+50%	Nominal	5	C2: +20% C1: +20%	4
Short	C2: Short R2: Short	3	C2: Short	3

TABLE VI-1 (Continued)

hits or possible hits could have been accomplished. Thus, using the conventional procedure, within tolerance variations introduced 17% to 42% errors.

B. RESULTS USING SENSITIVITY FUNCTIONS

Using the frequencies determined from the sensitivity functions a set of signatures was generated for the six circuit conditions. In order to check the effect of variations within tolerance for this method the same procedure as used in VI-A was followed. The result was 50 hits, 16 possible hits and 6 misses. The 6 misses included 3 due to the test signature showing normal response from the circuit. These results showed an error rate of 11% - 31%.

Eight additional circuit conditions were simulated for each element to see what could be expected from a less artificial situation. Table VI-2 lists all the conditions set and the numerical result. The numerical result was assigned as follows:

1 - closest library signature selected
2 - right element selected
3 - one of multiple signature closest to given condition
4 - one of multiple signatures right element
5 - incorrect selection.

Hits, misses and possible hits were also assigned as before. The result was 115 hits, 43 possible hits and 10 misses. Or, 68% hits and 94% hits or possible hits.

C. USING SUBROUTINE DET FOR CORRELATION

The same set of test signatures used in VI-B was used to test the programmed correlation and selection. The input and results are listed in Table VI-3. The results are coded as follows:

Circuit Condition	Run 1 V_2/V_1	Run 1 V_1/I_1	Run 2 V_2/V_1	Run 2 V_1/I_1
R1: Short	1	1	1	1
-90%	1	1	1	1
-70%	1	1	1	1
-50%	1	1	1	1
-40%	1	1	1	1
-20%	1	5	3	1
+20%	1	3	1	3
+30%	1	1	1	1
+50%	1	2	1	1
+60%	1	1	1	1
+90%	1	1	1	1
x3	1	1	1	1
X75	1	1	1	1
Open	1	1	1	1
L1: Short	1	1	1	1
-90%	1	1	1	1
-70%	1	1	2	2
-50%	1	1	1	1
-40%	1	1	1	1
-20%	1	1	1	1
+20%	1	5	1	5
+30%	1	3	1	1
+50%	1	1	1	1
+60%	1	1	1	1
+90%	1	1	1	1
X3	1	1	1	1
X75	1	1	1	1
Open	1	1	1	1
R2: Short	3	3	3	3
-90%	3	5	3	2
-70%	3	5	3	5
-50%	1	1	1	1
-40%	1	1	1	1
-20%	1	1	1	1
+20%	1	1	1	1
+30%	1	1	1	1
+50%	1	1	1	1
+60%	1	1	1	1
+90%	1	1	1	1
Open	1	1	1	1

TABLE VI-2. Results Using Sensitivity Functions.

C1: Open	1	1	1	1
-90%	5	1	5	1
-70%	5	1	5	1
-50%	3	1	5	1
-40%	3	1	3	1
-20%	1	3	5	3
+20%	4	5	5	1
+30%	3	1	5	1
+50%	5	1	5	1
+60%	3	1	5	1
+90%	4	1	3	1
X3	5	5	5	4
X75	1	1	1	1
Short	1	1	1	1
L2: Short	2	1	2	1
-90%	1	5	1	5
-70%	1	1	1	5
-50%	1	4	2	3
-40%	1	3	1	3
-20%	2	5	3	3
+20%	3	3	4	3
+30%	5	3	5	3
+50%	5	1	3	1
+60%	5	1	5	1
+90%	5	1	5	1
X3	5	1	5	1
X75	5	1	5	1
Open	1	1	5	1
C2: Open	1	1	1	1
-90%	1	1	5	1
-70%	1	1	5	1
-50%	3	1	4	1
-40%	3	1	3	3
-20%	5	3	5	3
+20%	5	3	5	3
+30%	3	1	5	1
+50%	5	1	5	1
+60%	3	1	3	1
+90%	3	1	5	1
X3	5	1	5	1
X75	3	3	3	3
Short	3	3	5	3

TABLE VI-2 (Continued)

1 - correct primary selection

2 - one of two primary selections count

3 - one of two secondary (no primary selections)

4 - incorrect primary selection.

The results were 61 rated 1, 12 rated 2, 21 rated 3 and 2 rated 4. Or, 63% completely correct and 98% with one of two selections correct. This is roughly the same result using manual correlation. The algorithm used for determining primary and secondary selection is outlined in Section VII-C.

Circuit Condition	V_2/V_1	V_1/I_1
R1		
-90%	1	1
-70%	1	1
-60%	1	1
+30%	1	1
+60%	1	1
+90%	1	1
X3	1	1
X75	1	1
L1		
-90%	1	1
-70%	1	1
-60%	1	1
+30%	2	2
+60%	1	1
+90%	1	1
X3	1	1
X75	1	1
C1		
-90%	3	3
-70%	1	1
-60%	3	3
+30%	3	2
+60%	3	2
+90%	1	1
X3	4	4
X75	1	1
R2		
-90%	1	1
-70%	1	1
-60%	1	1
+30%	1	1
+60%	1	1
+90%	1	1
X3	1	1
X75	1	1

TABLE VI-3. Computer Correlation Results.

L2		
-90%	3	3
-70%	3	1
-60%	1	1
+30%	2	2
+60%	2	2
+90%	3	3
X3	3	3
X75	3	3
C2		
-90%	3	3
-70%	2	2
-60%	3	3
+30%	1	1
+60%	1	1
+90%	1	1
X3	3	3
X75	2	2

TABLE VI-3 (continued)

VII. COMPUTER PROGRAMS

Several computer programs were written in Fortran IV to accomplish much of the mathematical work associated with this study. Four of the programs are included herein with detailed descriptions. Each of these programs were used with several modifications. However, only the basic program is presented in each instance. All listings are in the designated section.

A. PRIMARY ANALYSIS PROGRAM

The primary analysis program contains the main program and three subroutines. A fourth subroutine, PRQD, is used for root finding and is not included in the listing.

The program calculates the magnitude of both network functions as a function of the complex frequency, s, finds the roots of the polynomials involved and selects test frequencies.

Each element in turn is then set to each out-of-tolerance value as selected previously, the magnitude of each function at each test frequency is calculated and all responses are quantized into signatures.

The output consists of both network functions, the test frequencies, the nominal response at each test frequency and the list of library signatures.

The listing is the program which calculates the test frequencies as explained in Section IV-A. To modify the program for the selection from sensitivity functions the portion of the program between indicated comment cards was replaced with a read statement and the test frequencies were used as the input.

Figure 21 is the flow graph for the main analysis program.

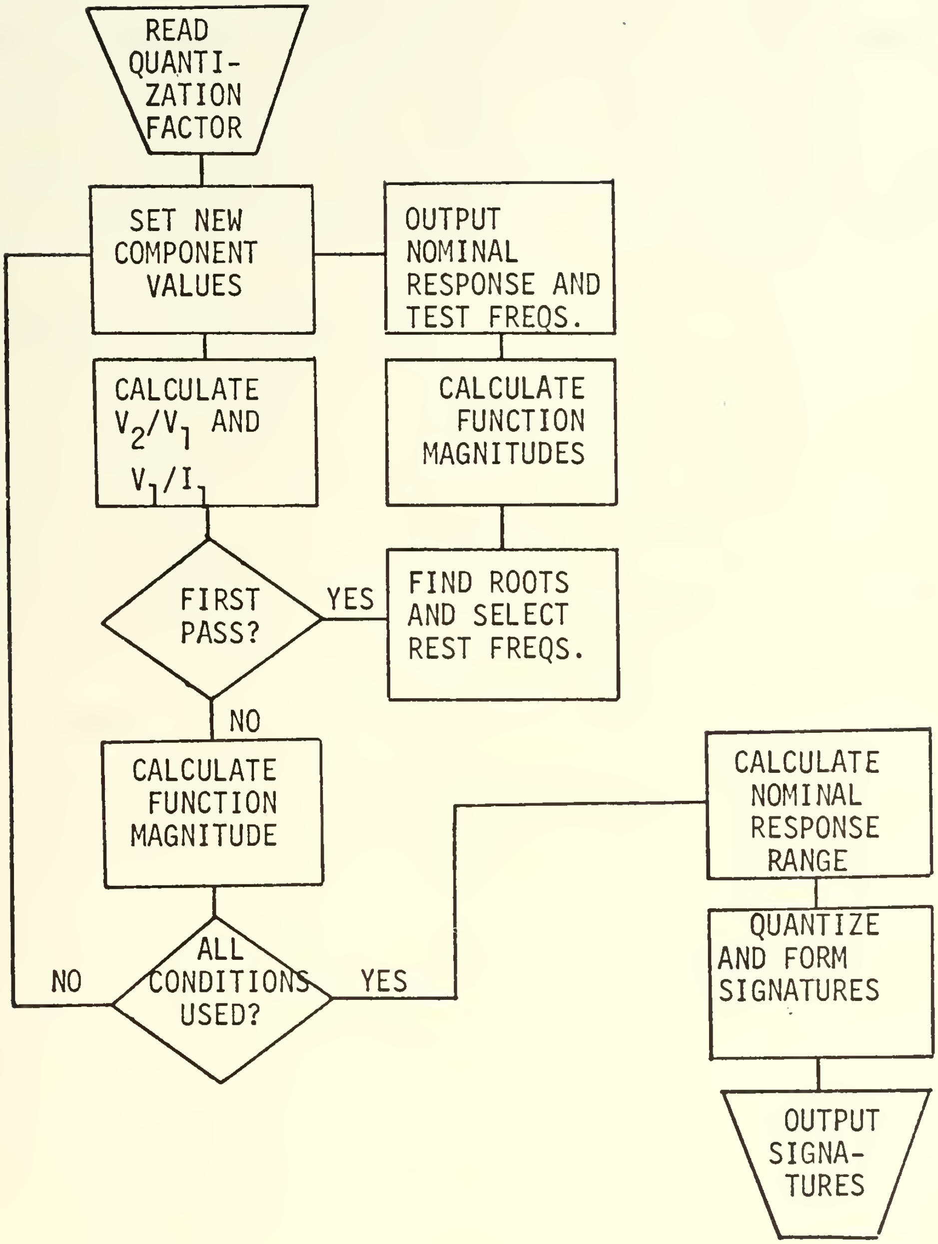

Figure 21. Flowgraph of Primary Analysis Program

B. TWO COMPONENT VARIATION PROGRAM

The main analysis program was modified to generate signatures for simultaneous two-component failure. Two different modifications were made, one generated signatures for only short and open element failures. The other allowed for short, -50%, +50% and open failures.

Since the programs were only modifications to the main analysis program flow data are not included and the listings are omitted.

C. FAULTY COMPONENT SELECTION

This program was written to determine possible and probable component failures from the two input signatures. The signature from V_1/I_1 and V_2/V_1 are both processed. A component is flagged as possibly failed if one of the library signatures for that component is selected as a match. If two or more library signatures for a component are selected as a match that component is flagged as probably failed.

The flow graph for the program is shown in Figures 22-24. The main program allows for several signatures to be read in and sent to subroutine DET, one pair at a time.

Subroutine DET first checks to see if the signature indicates the circuit is functioning properly (i.e., signature is all 5s). If so, a message is printed and control is returned to the main program. The first entry into DET with a non-nominal signature results in the library signatures being read in and stored.

When it has been determined that non-nominal signatures are being processed, both input signatures are compared with the library signatures to determine the closest match distance. The signatures are compared a second time and those library signatures that are the minimum distance

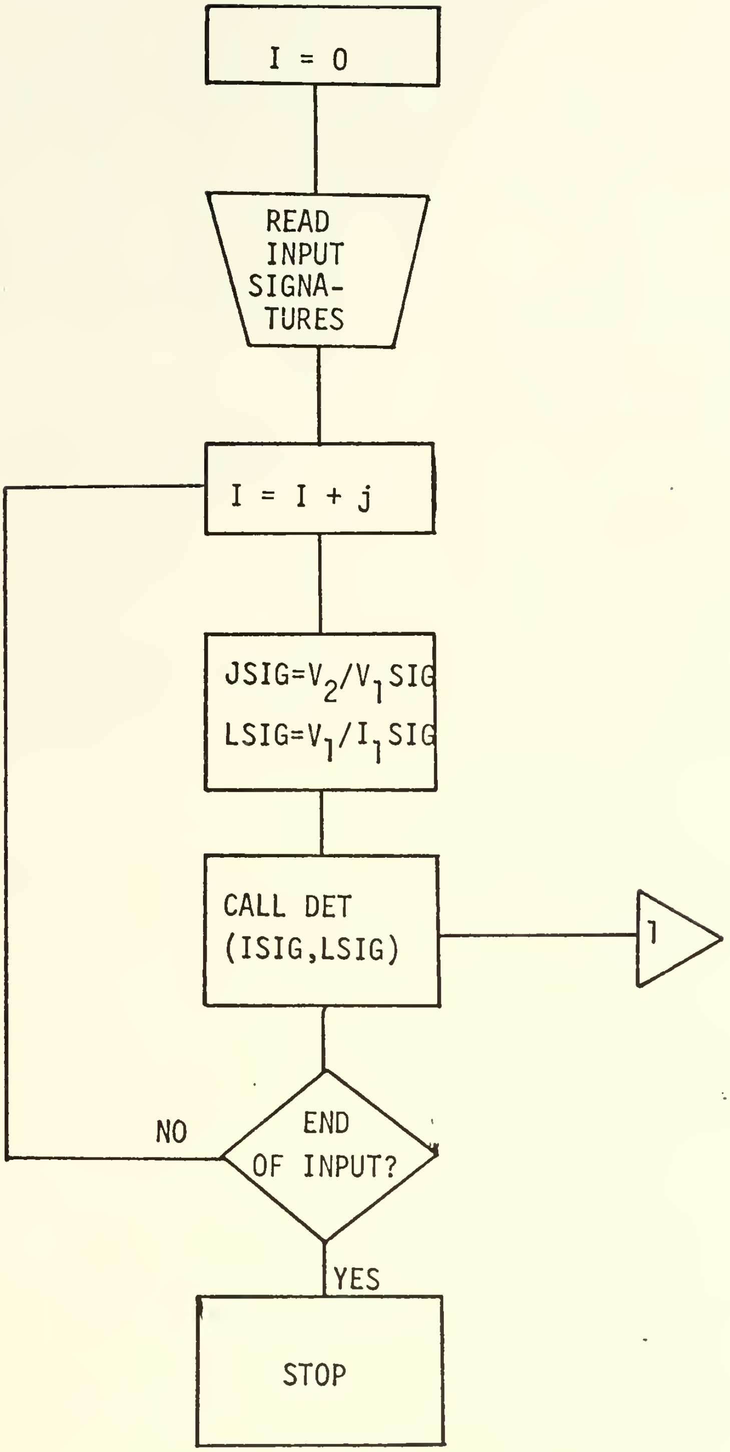

Figure 22. Flowgraph of Main Program for DET

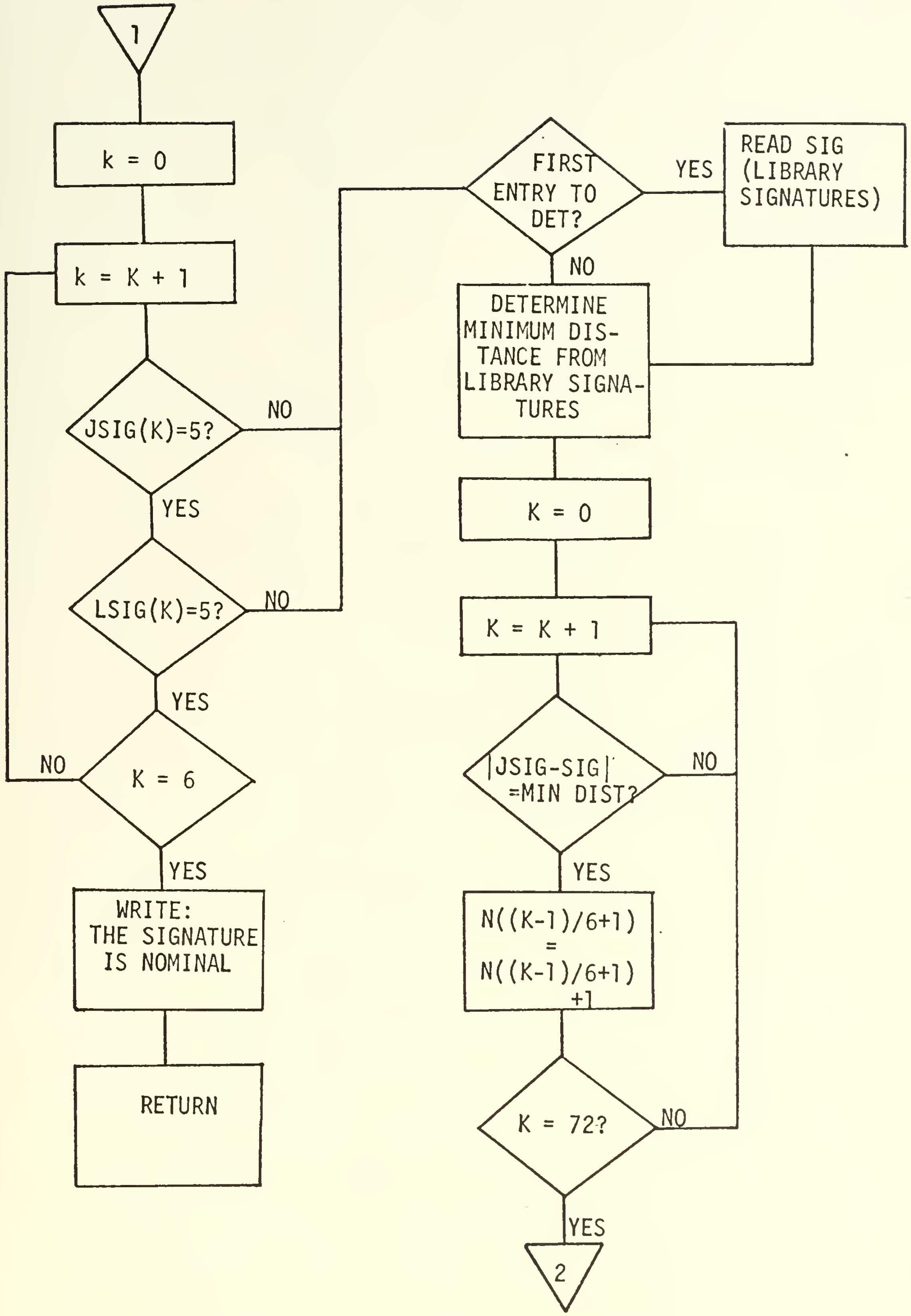

Figure 23. Flowgraph of Subroutine DET

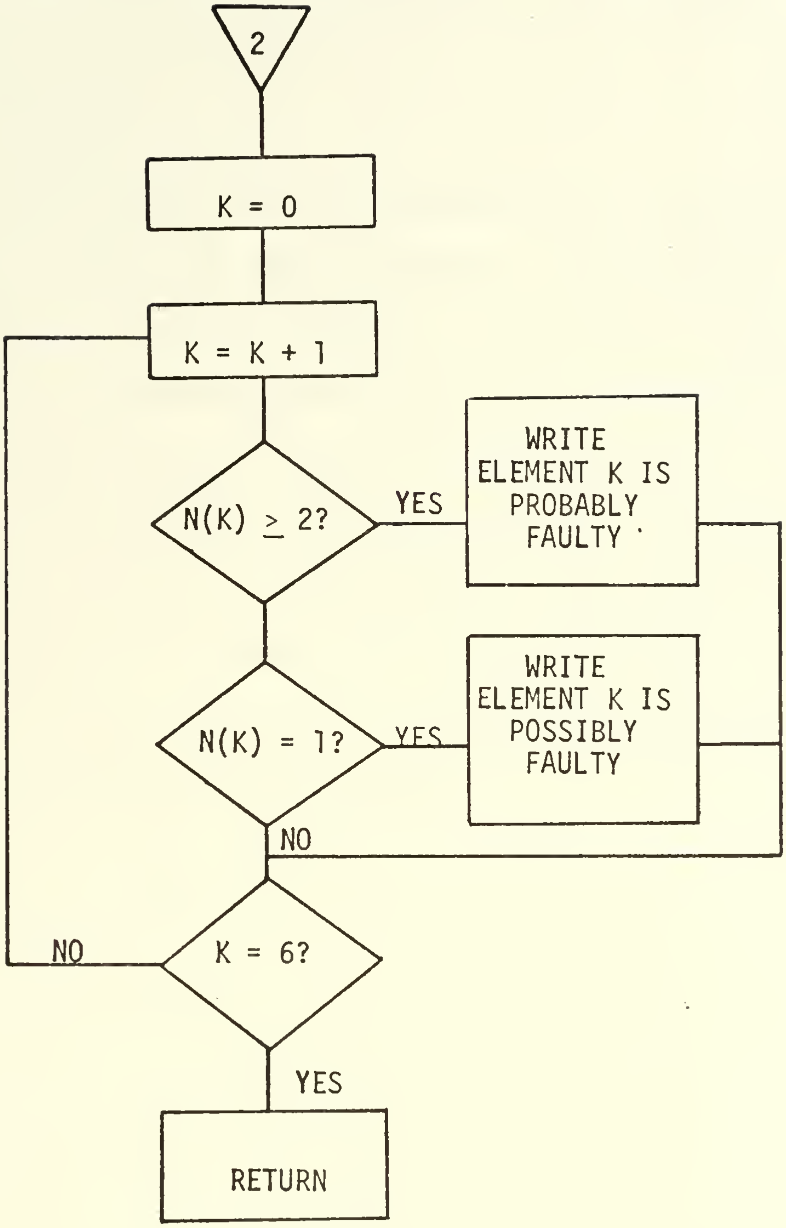

Figure 24. Flowgraph of Subroutine DET (Continued)

from the input are used to set flags for determining possible and probable faulty components as described earlier.

The program is listed showing the nearest neighbor matching procedure.

D. SUBROUTINE LSQ

Subroutine LSQ was written to determine which library signatures most closely matched the input signature. LSQ is similar in instruction flow to subroutine DET and therefore a flow chart is not presented.

In order to determine a match, LSQ uses a method similar to the nearest neighbor concept. However, if one considers the signature to be a vector with each test frequency a dimension, LSQ does not weigh every dimension equally. If one dimension can be strongly affected by random variations in component values that dimension is not considered equally with the other dimensions. Thus, the distance between the input vector and the library vectors becomes:

$$\sum_{i=1}^{n} a_i (X_i - Y_i)^2$$

where

$$\vec{X} = b_1 X_1 + b_2 X_2 + b_3 X_3 + \cdots$$
$$= \text{input vector}$$
$$\vec{Y} = c_1 Y_1 + c_2 Y_2 + c_3 Y_3 + \cdots$$
$$= \text{library vector}$$

a_i = weighting factor for the i^{th} dimension (or frequency)

$a_i \leq 1.$

LSQ operates on only one signature at a time and selects the proper library signature to match the input. The output is a list of the possible signatures and the component variation defined by each one.

One may choose to set each $a_i = 1$. This would be the case if the quantization process was sufficient to take care of the random variation problem.

VIII. SUGGESTED FURTHER STUDIES

A. ALTERNATE APPROACH TO SIGNATURE TABLE

Table VIII-1 lists the calculated response at each test frequency for the primary test circuit. Table VIII-2 lists the range of the response at each frequency and the maximum deviation from nominal response. Zero output is not included.

Test Frequency (Hz)	Response V_2/V_1	Response V_1/I_1
.00048	-2.28 Db	12.86 Ohms
.027	-6.77 Db	3.22 Ohms
.055	-10.85 Db	14.32 Ohms
.299	-56.8 Db	3.01 Ohms
2.99	-120.3 Db	4.34 Ohms
49.97	-215.5 Db	53.46 Ohms

TABLE VIII-1. Nominal Response at Test Frequencies

Test Frequency (Hz)	Response Range V_2/V_1 (db)		Response Range V_1/I_1 (Ω)		Maximum Deviation V_2/V_1 (db)	Maximum Deviation V_1/I_1 (Ω)
.00048	.0028	-29.91	3.0	66.77	-27.63	+53.91
.027	15.58	-46.16	.246	300.2	-39.39	+296.98
.055	-4.36	-55.7	3.0	302.	-44.85	+287.68
.299	-20.94	-97.14	.23	300.	-40.34	+296.99
2.99	-52.28	-160.41	3.	319.6	+68.02	+315.26
49.97	-98.48	-255.5	3.	5338.	+117.02	+5284.54

TABLE VIII-2. Response Range and Maximum Deviation

One can see that the responses do not cover the same range of values nor are the maximum deviation from the nominal response the same. From the nature of the network functions it can be assumed that the responses at different test frequencies are not independent for a given circuit condition. Due to these situations it would seem logical to weigh equal changes at different frequencies differently.

The setting of quantization levels and subsequent signature generation is one way to accomplish this weighing. The setting of quantization levels based on the selected situations tends to weigh responses at some test frequencies less than others. However, this is purely an "educated guess" method and has no analytical foundation.

An alternate scheme was studied briefly and is suggested as the subject of further study.

If one considers each test frequency as one of the dimensions in an n-dimensional space then each response would be a point in the n-dimensional space. Instead of limiting calculations to a few selected circuit conditions one could generate many points representing many circuit conditions. One could then have a large enough sample of data to do some statistical operations.

In order to remove dependence the normal procedure is to multiply each vector, representing a point, by the inverse of the co-variance matrix. The result is the collection of points in a "transformed space" where dependency is accounted for.

The problem is the construction of the co-variance matrix. However, since the data base is now sufficiently large,an estimate of the matrix can be calculated from the data.

With a circuit under test the response would constitute a vector which would be multiplied by the inverse of the covariance matrix. At this point there are several options for fault identification. One can make a comparison with the stored data and make the fault identification using the nearest neighbor technique. An alternate approach would be to use an average of the nearest m neighbors when m is some positive integer. There are other possible procedures for matching.

If one is only interested in which component is faulty it might also be of interest to investigate any possible grouping of points which would be useful as in pattern recognition. The unknown could then be assigned to a group according to its position. Reference 5 details the necessary pattern recognition information.

The above described procedure would require a computer with sufficient storage to process and store the information pertaining to the data base and covariance matrix. However such a computer is readily available for a nominal cost.

B. EXTENSION OF PROCEDURE

It soon becomes obvious that for even moderate sized circuits the procedure would be very difficult to use. For example, a 25-element circuit with 6 possible element value perturbations would yield 150 signatures. For ease in testing the number of test frequencies should be as small as possible. However, with 150 signatures considerable signature duplication is probable unless the number of test frequencies is large.

Reference 3 shows how a ladder network may be divided into subnetworks such that the voltage gain of a given subnetwork is independent of the voltage gain of all preceding networks. Therefore, testing by adding subnetworks one at a time can yield the fault component.

This procedure uses only the voltage gain but does show one approach to handing large circuits. In any case, a large network would have to be divided into subnetworks by some scheme in order to reduce the problem to a manageable size.

IX. CONCLUSIONS

The majority of the conclusions made as a result of this study were based on a comparison of the results is section VI and results published by other investigators.

Most investigators reported approximately 75% isolation using one network function with no allowance for variation-within-tolerance of the nonfaulty elements. One investigator [2] reported 78%-98% success. However, the results were not sufficiently documented to judge the merit of the claim.

The results of this study show that an additional error rate of 17%-42% can be introduced by the within tolerance variations if the conventional procedures are followed. However, if sensitivity functions are used to aid in selecting test frequencies the error rate due to other element variations drops to 11%-31% for one network function. Furthermore, using two network functions fault isolation is successful for up to 94% of random single fault conditions.

The high degree of successful fault isolation clearly points out the advisability of using at least two network functions for testing. The two functions used for this study were V_2/V_1 and V_1/I_1. However, the designed purpose of the circuit may dictate a different choice of functions to be used.

The results clearly show the improved accuracy of isolation using sensitivity functions as a guide in test frequency selection. In addition to improved isolation this method also eliminates the possibility of unknowingly picking test frequencies which yield the same information.

VAR	V2/V1	V1/I1
	ELEMENT 1 R1	
1	9 9 9 9 7 9 7	3 2 1 1 2 1 3
2	9 9 9 9 6 9 6	9 4 2 2 3 2 4
3	7 7 7 7 5 6 5	5 5 4 4 9 4
4	4 4 4 4 5 9 5	5 5 7 7 5 7
5	3 3 3 3 9 3 9	7 7 9 9 8 9 8
6	1 1 1 1 2 1 1	9 9 9 9 9 9 9
	ELEMENT 2 L1	
1	5 5 5 5 5 5 9	5 5 5 5 5 9 2
2	5 5 5 5 5 5 8	5 5 5 5 5 5 2
3	5 5 5 5 5 5 6	5 5 5 5 5 5 4
4	5 5 5 5 5 5 9	5 5 5 5 5 5 7
5	5 5 5 5 5 9 4	5 5 5 5 5 6 9
6	5 5 7 3 4 1 1	5 5 4 9 9 9 9
	ELEMENT 3 C1	
1	5 5 5 9 5 9 9	5 5 7 5 8 9 9
2	5 5 5 5 7 9 9	5 5 6 5 9 5 5
3	5 5 5 5 6 7 7	5 5 5 5 8 5 5
4	5 5 5 5 9 4 4	5 5 5 5 4 5 5
5	5 5 5 5 4 3 3	5 5 9 5 3 5 5
6	5 1 1 1 3 1 1	5 1 1 5 2 5 5
	ELEMENT 4 R2	
1	1 1 1 1 3 1 1	1 1 1 8 5 5 5
2	2 2 3 5 5 5 5	2 2 5 5 5 5 5
3	4 4 9 5 5 5 5	4 4 5 5 5 5 5
4	6 6 5 5 5 5 5	7 7 5 5 5 5 5
5	8 8 6 5 5 5 5	9 9 5 5 5 5 5
6	9 9 8 5 5 5 5	9 9 4 5 5 5 5
	ELEMENT 5 L2	
1	5 5 9 3 8 9 9	5 5 5 5 2 5 5
2	5 5 5 9 9 9 9	5 5 5 5 9 5 5
3	5 5 5 5 7 7 7	5 5 5 5 6 5 5
4	5 5 5 5 4 4 4	5 5 5 5 5 5 5
5	5 5 5 5 3 3 3	5 5 5 7 9 5 5
6	5 4 1 1 2 1 1	5 8 9 9 3 5 5
	ELEMENT 6 C2	
1	5 5 7 9 9 9 9	5 7 9 9 4 5 5
2	5 5 6 8 9 9 9	5 6 9 5 5 5 5
3	5 5 5 6 6 7 7	5 5 9 5 5 5 5
4	5 5 5 9 9 4 4	5 5 4 5 5 5 5
5	5 5 4 3 4 3 3	5 9 3 5 5 5 5
6	5 1 1 1 2 1 1	5 1 1 8 5 5 5

Figure 25. Signatures from Basic Procedure

VAR	V2/V1						V1/I1					
			ELEMENT 1				R1					
1	3	1	5	1	3	5	1	1	5	2	4	5
2	4	3	5	4	3	5	3	1	5	4	4	5
3	5	5	5	4	4	5	4	4	5	4	5	5
4	5	5	5	6	5	5	6	6	5	6	5	5
5	6	6	5	6	6	5	7	6	5	6	6	5
6	9	9	9	9	9	9	9	9	8	9	9	7
			ELEMENT 2				L1					
1	5	5	5	5	3	1	5	5	5	5	4	3
2	5	5	5	5	3	3	5	5	5	5	4	4
3	5	5	5	5	4	4	5	5	5	5	5	4
4	5	5	5	5	5	6	5	5	5	5	5	6
5	5	5	5	5	7	7	5	5	5	5	6	6
6	5	6	5	8	9	9	5	6	6	8	9	9
			ELEMENT 3				C1					
1	5	5	2	8	9	9	5	5	7	3	1	2
2	5	5	3	5	5	5	5	5	6	4	4	4
3	5	5	4	5	5	5	5	5	5	4	4	4
4	5	5	9	5	5	5	5	5	5	6	5	6
5	5	5	5	5	5	5	5	5	5	6	6	6
6	2	5	1	5	5	5	7	8	9	9	9	9
			ELEMENT 4				R2					
1	2	5	6	5	5	5	9	9	9	9	9	9
2	3	5	5	5	5	5	7	5	5	5	5	5
3	4	5	5	5	5	5	6	5	5	5	5	5
4	6	5	5	5	5	5	4	5	5	5	5	5
5	7	5	5	5	5	5	4	5	5	5	5	5
6	8	5	5	5	5	5	1	5	5	5	5	5
			ELEMENT 5				L2					
1	5	5	1	5	5	5	5	6	7	2	1	1
2	5	5	2	5	5	5	5	5	5	4	4	4
3	5	5	3	5	5	5	5	5	4	4	4	4
4	5	5	9	5	5	5	5	5	6	6	5	6
5	5	5	5	5	5	5	5	5	7	6	6	6
6	5	7	2	5	5	5	5	9	9	9	9	9
			ELEMENT 6				C2					
1	5	8	2	5	5	5	5	2	2	2	1	2
2	5	5	3	5	5	5	5	3	2	4	4	4
3	5	5	5	5	5	5	5	5	4	4	4	4
4	5	5	5	5	5	5	5	5	6	6	5	6
5	5	5	5	5	5	5	5	6	7	6	6	6
6	2	5	6	5	5	5	9	9	9	9	9	9

Figure 26. Signatures using Sensitivity Procedure

V2/V1 V1/I1

THE VARIED ELEMENTS ARE 1 AND 2

6	6	6	6	1	6	6	4	4	3	3	4	3	2
6	6	6	6	1	6	5	4	4	3	3	4	3	5
3	3	3	2	4	4	1	1	1	6	6	6	1	3
3	3	3	2	4	2	2	1	1	6	6	1	6	7

THE VARIED ELEMENTS ARE 1 AND 3

6	6	6	6	6	6	6	4	4	3	3	4	3	4
6	6	6	6	4	3	3	4	4	3	3	1	3	4
3	2	2	2	5	5	1	1	1	6	6	6	6	1
3	3	3	2	4	1	2	1	1	6	6	1	6	1

THE VARIED ELEMENTS ARE 1 AND 4

5	5	6	6	1	6	1	3	3	3	3	4	3	4
7	7	7	6	1	6	1	6	6	3	3	4	3	4
2	2	3	2	4	3	4	3	4	6	6	1	6	1
5	5	4	3	4	3	4	6	7	6	6	1	6	1

THE VARIED ELEMENTS ARE 1 AND 5

6	6	6	1	6	6	6	4	4	3	3	4	3	4
6	6	6	6	2	3	3	4	4	3	3	1	3	4
3	3	3	3	1	5	1	1	1	6	1	6	6	1
3	3	3	3	2	1	2	1	1	6	6	1	6	1

THE VARIED ELEMENTS ARE 1 AND 6

6	6	6	6	6	6	6	4	5	4	3	4	3	4
6	6	1	5	3	3	3	4	3	2	3	4	3	4
3	3	3	4	1	5	1	1	6	7	6	6	6	1
3	3	3	1	2	1	2	1	5	3	6	1	6	1

THE VARIED ELEMENTS ARE 2 AND 3

5	5	5	5	1	6	6	5	5	1	5	1	4	3
5	5	5	5	4	2	3	5	5	4	5	1	4	3
5	5	5	5	1	6	5	5	5	1	5	1	5	6
5	5	5	5	4	2	1	5	5	4	5	1	5	6

THE VARIED ELEMENTS ARE 2 AND 4

3	3	4	5	5	1	1	3	3	5	5	5	4	3
7	7	6	5	5	1	1	6	6	5	5	5	4	3
3	3	4	5	5	5	3	3	3	5	5	5	5	6
7	7	6	5	5	5	3	6	6	5	5	5	5	6

THE VARIED ELEMENTS ARE 2 AND 5

5	5	5	5	6	6	6	5	5	5	4	1	4	3
5	5	5	1	2	2	3	5	5	5	5	1	4	3
5	5	5	5	6	6	5	5	5	5	4	1	5	6
5	5	5	1	2	2	1	5	5	5	5	1	5	6

THE VARIED ELEMENTS ARE 2 AND 6

5	5	1	6	1	6	6	5	1	6	5	5	4	3
5	5	4	3	2	2	3	5	4	2	5	5	4	3
5	5	1	6	1	6	5	5	1	6	5	5	5	6
5	5	4	3	2	2	1	5	4	2	5	5	5	6

Figure 27. Signatures for Two Component Variations

V2/V1								V1/I1						

THE VARIED ELEMENTS ARE I AND 2

6	6	6	6	1	6	6		9	9	1	7	1	1	9
6	6	6	6	6	7	7		4	4	3	3	4	3	2
6	6	6	6	1	6	5		4	4	3	2	2	1	1
6	6	6	6	1	1	4		4	4	3	3	4	3	5
9	8	7	7	7	1	9		9	8	7	8	7	8	7
9	9	8	7	7	7	7		4	4	3	3	4	3	1
9	9	8	7	7	7	7		1	1	1	1	1	1	1
9	9	8	7	7	7	7		1	1	1	1	1	1	1
9	9	8	7	7	7	7		1	1	1	1	1	1	1
9	9	8	7	7	7	1		1	1	1	1	1	1	1
3	3	3	2	4	4	1		7	8	8	7	8	7	8
3	3	2	1	1	7	7		1	1	6	6	6	1	3
3	3	3	2	4	2	2		1	1	6	3	3	1	1
3	3	3	2	4	2	1		1	1	6	6	1	6	7
9	9	8	7	7	7	7		1	1	6	6	1	6	7
2	2	2	1	4	3	1		7	8	8	7	8	7	8
2	2	1	1	1	7	7		1	6	7	7	6	6	3
2	2	2	1	4	1	2		1	6	7	4	3	1	1
2	2	2	1	4	1	1		1	6	7	7	6	7	7
2	2	2	1	4	1	1		1	6	7	7	6	7	8
9	9	1	7	1	9	7		7	8	8	7	8	7	8
9	9	8	7	1	7	1		7	8	8	7	8	7	8
9	9	8	7	1	7	1		7	7	8	7	8	7	8
9	1	1	1	7	7	1		7	8	8	7	8	7	8
9	9	8	1	1	7	1		7	8	8	7	8	7	8

THE VARIED ELEMENTS ARE I AND 3

9	9	8	7	7	7	9		7	8	8	7	8	7	8
6	6	6	6	6	6	6		4	4	3	3	4	3	4
6	9	8	7	7	7	7		4	7	6	3	5	3	4
6	6	6	6	4	3	3		4	4	3	3	1	3	4
9	1	7	7	7	1	9		8	8	7	8	7	8	7
6	6	6	6	5	1	1		4	4	3	3	3	1	1
9	9	8	7	7	7	7		1	1	1	1	1	1	1
9	9	8	7	7	7	7		1	1	1	1	1	1	1
9	9	8	7	7	7	7		1	1	1	1	1	1	1
9	9	8	7	7	4	2		1	1	1	1	1	1	1
9	9	8	7	1	1	7		7	8	8	7	8	7	8
3	3	3	2	5	5	1		1	1	6	6	6	6	1
3	8	8	7	7	7	7		1	8	8	6	7	6	1
3	3	3	2	4	1	2		1	1	6	6	1	6	1
3	3	3	3	4	1	1		1	1	1	6	4	1	7
9	9	8	9	1	1	1		7	8	8	7	8	7	8
2	2	2	1	4	3	1		1	6	7	7	7	7	1
2	2	1	1	6	7	7		1	8	8	7	8	6	1
2	2	2	1	4	1	2		1	6	6	7	1	7	1
2	2	2	1	4	1	1		1	6	6	7	5	1	8
9	9	8	7	1	7	1		7	8	8	7	8	7	8
9	9	1	7	7	7	1		7	8	8	7	8	7	8
9	1	8	7	1	1	7		7	8	9	7	8	7	8
9	1	8	1	1	7	1		7	8	8	7	8	7	8
9	9	8	7	1	7	7		7	8	8	7	8	7	8

Figure 27 (Continued)

```
      REAL*8 WMEGA(320)
      REAL MAG,NMAG,WMAG(2,320)
      DIMENSION OMEGA (5,5),E(10),D(5),Q(5),R(5),F(5),UN(5),
     1Y(5),EI(5),Q1(10),MAG(4,10),NMAG(6,10,20),DX(5),YX(5),
     2Z(5),XMAX(20),XMIN(20),INMAG(6,10,20),DMAG(2,10)
      COMMON NMAG
      DATA OMEGA/25*0./,Q/5*0./,R/5*0./,EI/5*0./,F/5*0./
      DATA Q1/10*0./
      READ (5,110) FACO
      E(1)=3.
      E(2)=.17
      E(3)=1.000001
      E(4)=10.00001
      E(5)=9.
      E(6)=4.
      DO 38 I=1,6
      DO 38 K=1,10
      GO TO (1,2,3,4,5,6,7,8,9,10),K
    1 TEMP=E(I)
      GO TO 11
    2 E(I)=1.1*TEMP
      GO TO 11
    3 E(I)=0.9*TEMP
      GO TO 11
    4 E(I)=0.
      GO TO 11
    5 E(I)=0.5*TEMP
      GO TO 11
    6 E(I)=0.8*TEMP
      GO TO 11
    7 E(I)=1.2*TEMP
      GO TO 11
    8 E(I)=1.5*TEMP
      GO TO 11
    9 E(I)=100.*TEMP
      GO TO 11
   10 E(I)=TEMP
      GO TO 38
   11 UN(1)=E(4)
      D(1)=E(1)+E(4)
      D(2)=E(2)+E(5)+E(1)*E(4)*(E(3)+E(6))
      D(3)=E(5)*(E(1)*E(3)+E(4)*E(6))+E(4)*E(2)*(E(3)+E(6))
      D(4)=E(5)*(E(1)*E(4)*E(3)*E(6)+E(2)*E(3))
      D(5)=E(2)*E(3)*E(4)*E(5)*E(6)
      Y(1)=1.
      Y(2)=E(4)*(E(3)+E(6))
      Y(3)=E(3)*E(5)
      Y(4)=E(3)*E(4)*E(5)*E(6)
      IF (K.GT.1.OR.I.GT.1) GO TO 36
CC    WRITE (8,100) UN(1)
CC    WRITE (8,101)
CC    WRITE (8,102) (D(N),N=1,5)
CC    WRITE (8,102) (D(N),N=1,5)
CC    WRITE (8,104)
CC    WRITE (8,103) (Y(N),N=1,4)
C ---- DELETE STARTING HERE FOR SENSITIVITY FUNCTION USE
      DO 1111 LM=1,5
      DX(LM)=D(LM)
 1111 YX(LM)=Y(LM)
      CALL PROD (DX,5,Q,EI,PCL,IR,IER)
      IF (IER.EQ.0) GO TO 12
CC    WRITE (8,105) IER
CC 12 WRITE (8,106) (Q(N),EI(N),N=1,4)
   12 CONTINUE
      CALL PROD (YX,4,R,F,POM,IR,IER)
      IF (IER.EQ.0) GO TO 13
CC    WRITE (8,105) IER
CC 13 WRITE (8,106) (R(N),F(N),N=1,3)
   13 CONTINUE
      DO 15 M=1,4
      IF (ABS(EI(M)).LE.1E-10) GO TO 14
      OMEGA (1,M)=ABS(EI(M))
```

```
      GO TO 15
   14 OMEGA (1,M)=ABS(Q(M))
   15 CONTINUE
      DO 17 M=1,3
      IF (ABS(F(M)).LE.1E-10) GO TO 16
      OMEGA (2,M)=ABS(F(M))
      GO TO 17
   16 OMEGA (2,M)=ABS(R(M))
   17 CONTINUE
      DO 18 M=1,2
      DO 18 N=1,4
      DO 18 NO=1,4
      IF(OMEGA(M,N).EQ.OMEGA(M,NO).AND.N.NE.NO) GO TO 75
      GO TO 18
   75 OMEGA (M,N)=0.
   18 CONTINUE
      DO 21 J=1,4
      DO 20 L=J,4
      IF (OMEGA(1,J).GT.OMEGA(1,L)) GO TO 19
      GO TO 20
   19 OMEGA (1,5)=OMEGA (1,J)
      OMEGA (1,J)=OMEGA (1,L)
      OMEGA (1,L)=OMEGA (1,5)
   20 CONTINUE
   21 CONTINUE
      DO 22 L=1,4
      IF (OMEGA(1,L).EQ.0.) GO TO 22
      LP=L+1
      OMEGA (3,5)=(OMEGA(1,LP)-OMEGA(1,L))/2.
      OMEGA (3,1)=OMEGA (1,L)-OMEGA (3,5)
      OMEGA (3,2)=OMEGA (1,L)+OMEGA (3,5)
      IF (OMEGA (3,1).LT.0.) OMEGA (3,1)=OMEGA(1,L)/2.
      LP2=L+2
      IF (LP2.GT.4) GO TO 23
      OMEGA (3,5)=(OMEGA(1,LP2)-OMEGA(1,LP))/2.
      OMEGA (3,3)=OMEGA (1,LP)+OMEGA (3,5)
      OMEGA (3,4)=OMEGA (1,LP2)+OMEGA(3,5)
      IF (LP2.EQ.4) GO TO 23
      LP3=L+3
      OMEGA (3,5)=(OMEGA (1,LP3)-OMEGA (1,LP2))/2.
      OMEGA (3,4)=OMEGA (1,LP2)+OMEGA (3,5)
      OMEGA (3,5)=OMEGA(1,LP3)+OMEGA (3,5)
      GO TO 23
   22 CONTINUE
   23 DO 26 J=1,3
      DO 25 L=J,3
      IF (OMEGA (2,J).GT.OMEGA (2,L)) GO TO 24
      GO TO 25
   24 OMEGA (2,5)=OMEGA (2,J)
      OMEGA (2,J)=OMEGA (2,L)
      OMEGA (2,L)=OMEGA (2,5)
   25 CONTINUE
   26 CONTINUE
      DO 27 L=1,3
      IF (OMEGA (2,L).EQ.0.) GO TO 27
      LP=L+1
      OMEGA (4,5)=(OMEGA (2,LP)-OMEGA (2,L))/2.
      OMEGA (4,1)=OMEGA (2,L)-OMEGA (4,5)
      OMEGA (4,2)=OMEGA (2,L)+OMEGA (4,5)
      OMEGA (4,3)=OMEGA (2,LP)+OMEGA (4,5)
      IF (OMEGA(4,1).LT.0.) OMEGA (4,1)=OMEGA(2,L)/2.
      LP2=L+2
      IF (LP2.GT.3) GO TO 28
      OMEGA (4,5)=(OMEGA (2,LP2)-OMEGA (2,LP))/2.
      OMEGA (4,3)=OMEGA (2,LP)+OMEGA (4,5)
      OMEGA (4,4)=OMEGA (2,LP2)+OMEGA (4,5)
      GO TO 28
   27 CONTINUE
   28 M2=0
      DO 29 M=1,4
      DO 29 M1=3,4
      M2=M2+1
```

```
   29 O1(M2)=OMEGA(M1,M)
      O1(M2+1)=OMEGA(3,5)
      DO 291 IL=1,10
      DO 291 IJ=IL,10
      IF (O1(IJ).EQ.0.) GO TO 291
      OX=O1(IL)/O1(IJ)
      IF (OX.NE.1..AND.(OX.GT..9.AND.OX.LT.1.1)) O1(IL)=0.
  291 CONTINUE
      DO 32 M3=1,9
      DO 31 M4=M3,9
      IF (O1(M3).LE.O1(M4)) GO TO 31
      O1(10)=O1(M3)
      O1(M3)=O1(M4)
      O1(M4)=O1(10)
   31 CONTINUE
   32 CONTINUE
      DO 34 M5=1,9
      IF (O1(M5).EQ.0.) GO TO 34
      M6=10-M5
      IF (M5.EQ.1) GO TO 35
      DO 33 M7=1,M6
   33 O1(M7)=O1(M5+M7-1)
      GO TO 35
   34 CONTINUE
   35 O1(M6+1)=0.
      M6=M6+1
C ---- STOP DELETION HERE
   36 CALL MAG1 (M6,O1,UN,D,Y,MAG)
      IF (K.NE.1.OR.I.NE.1) GO TO 372
      DO 37 LM=1,10
      MAG (3,LM)=MAG(1,LM)
      MAG (4,LM)=MAG(2,LM)
   37 WRITE (4,115) MAG(3,LM),MAG(4,LM)
CC    WRITE (8,107) (O1(M8),M8=1,M6)
      WRITE (4,115) (O1(M8),M8=1,M6)
CC    WRITE (8,108)
CC    WRITE (8,109) (MAG(1,N),MAG(2,N),N=1,M6)
  372 DO 500 IK=1,2
      DO 500 IL=1,M6
      IF(MAG(IK,IL).LE.0.) GO TO 499
      DMAG(IK,IL)=20.*ALOG10(MAG(IK,IL))
      GO TO 500
  499 DMAG(IK,IL)=0.
  500 CONTINUE
      DO 38 LM=1,10
      IF (MAG(3,LM).LE.0.) GO TO 371
      NMAG(I,K,LM)=MAG(1,LM)/MAG(3,LM)
  371 IF(MAG(4,LM).LE.0.) GO TO 38
      NMAG(I,K,LM+10)=MAG(2,LM)/MAG(4,LM)
   38 CONTINUE
      DO 39  K=1,19
   39 CALL NOM(6,2,3,K,XMAX(K),XMIN(K))
      WRITE (4,115) (XMAX(KZ),XMIN(KZ),KZ=1,19)
      DO 40 K=1,9
   40 CALL QUANTA (K,XMIN(K),XMAX(K),FACQ)
      DO 41 K=11,19
   41 CALL QUANTA (K,XMIN(K),XMAX(K),FACQ)
      DO 411 I=1,6
      DO 411 J=1,10
      DO 411 K=1,20
  411 INMAG(I,J,K)=NMAG(I,J,K)
      DO 43 I=1,6
      DO 42 J=4,9
CC 42 WRITE (8,112) (INMAG(I,J,K),K=1,M6)
   42 CONTINUE
CC    WRITE (8,113)
   43 CONTINUE
      DO 45 I=1,6
      DO 44 J=4,9
      M6P=M6+10
CC 44 WRITE (8,112) (INMAG(I,J,K),K=11,M6P)
   44 CONTINUE
```

```
CC    WRITE (8,113)
   45 CONTINUE
      WRITE (4,112) (((INMAG(I,J,K),K=1,M6),J=4,9),I=1,6)
      STOP 1
  100 FORMAT (15X,1PE13.5)
  101 FORMAT (6X,'V2/V1 = ------------------------------------------
     1---------------------------------------------')
  102 FORMAT (15X,1PE13.5,1X,'+',1PE13.5,1X,'S**1  +'1PE13.5,
     11X,'S**2 + '
     1,1PE13.5,1X,'S**3 + ',1PE13.5,1X,'S**4',/)
  103 FORMAT (15X,1PE13.5,' + '1PE13.5,' S**1 + 'IPE13.5,' S
     1**2 + '1PE13.5,' S**3')
  104 FORMAT (6X,' Z1,1 = ------------------------------------------
     1------------------------------------')
  105 FORMAT (I10)
  106 FORMAT (////10X,'REAL PART IMAG PART REAL PART IMAG PA
     1RT REAL PART IMAG PART REAL PART IMAG PART'/9X,8F10.3)
  107 FORMAT (////1X,'THE FREQUENCIES USED ARE:',/1P10E12.4)
  108 FORMAT (//1X,'V2/V1 MAGNITUDE        Z1,1  MAGNITUDE'/)
  109 FORMAT (1PE15.5,1PE21.5)
  110 FORMAT (F5.0,I5)
  111 FORMAT (1X,1PD20.5,1P2E20.5)
  112 FORMAT (1X,7I4)
  113 FORMAT (1H )
  115 FORMAT (2E16.7)
  116 FORMAT (5E16.7)
      STOP
      END


      SUBROUTINE MAG1 (N,OMEGA,XNUM,DEN,YNUM,MAG)
      REAL MAG
      DIMENSION OMEGA(10),XNUM(5),DEN(5),YNUM(5),MAG(4,10)
      DO 1 I=1,N
      W=OMEGA(I)
      YR=YNUM(1)-YNUM(3)*W*W
      YI=YNUM(2)*W-YNUM(4)*W**3
      DR=DEN(1)-DEN(3)*W*W+DEN(5)*W**4
      DI=DEN(2)*W-DEN(4)*W**3
      YM=SQRT(YR*YR+YI*YI)
      DM=SQRT(DR*DR+DI*DI)
      MAG(1,I)=XNUM(1)/DM
    1 MAG(2,I)=DM/YM
      RETURN
      END


      SUBROUTINE QUANTA (K,QL,QH,FAC)
C   K=TERM IN ARRAY
C   QL=LOWER LIMIT FOR TOL.
C   QH=UPPER LIMIT FOR TOL.
C   FAC=FACTOR FOR QUANTIZATION
      REAL NMAG(6,10,20),Z(5)
      COMMON NMAG
      DO 5 I=1,6
      DO 5 J=1,10
      QU=NMAG(I,J,K)
      Z(1)=FAC
      Z(2)=FAC*3.
      Z(3)=FAC*6.
      Z(4)=FAC*12.
      IF (QU.LT.QL) GO TO 3
      IF (QU.GT.QH) GO TO 1
      NMAG(I,J,K)=0.
      GO TO 5
    1 DO 2 L=1,4
      QFAC=Z(L)*(QH-1.)+QH
      IF (QU.GT.QFAC) GO TO 2
      NMAG (I,J,K)=L
      GO TO 5
    2 NMAG (I,J,K)=5.
      GO TO 5
```

```
3 DO 4 M=1,4
  QFAC=QL-Z(M)*(1.-QL)
  IF (QU.LT.QFAC) GO TO 4
  NMAG (I,J,K)=M+4
  GO TO 5
4 NMAG(I,J,K)=10.
5 CONTINUE
  RETURN
  END

  SUBROUTINE NOM (I,J1,J2,K,XMAX,XMIN)
  REAL NMAG(6,10,20)
  COMMON NMAG
  XMAX=1.
  XMIN=1.
  DO 1 L=1,I
  DO 1 M=J1,J2
  IF (NMAG(L,M,K).GT.XMAX) XMAX=NMAG(L,M,K)
  IF (NMAG(L,M,K).LT.XMIN) XMIN=NMAG(L,M,K)
1 CONTINUE
  RETURN
  END
```

```
      DIMENSION ISIG(12,9,6),JSIG(6)
      DO 1 I=1,6
      DO 1 K=1,9
    1 READ(5,100) (ISIG(I,K,L),L=1,6),(ISIG(I+6,K,L),L=1,6)
      DO 4 I=1,12
      DO 4 J=1,9
      DO 3 K=1,6
    3 JSIG(K)=ISIG(I,J,K)
      WRITE(6,101) I,J,(JSIG(K1),K1=1,6)
      CALL LSQ(I,J,JSIG)
    4 CONTINUE
  100 FORMAT(12I2)
  101 FORMAT(/,5X,'THE INPUT IS: ELEMENT',I3,'  VAR. NO.',I2
      END
      SUBROUTINE LSQ(I1,J1,JSIG)
      REAL ID(36),IS
      DIMENSION ISIG(36,6),JSIG(6),KSIG(36,6),FAC(6)
      IF (I1-1) 1,6,1
    6 IF (J1-1) 1,9,1
    9 FAC(1)=1.
      FAC(2)=.3333
      FAC(2)=.3333;FAC(3)=.565;FAC(4)=.462;FAC(5)=.694
      FAC(6)=.745
      DO 2 J=1,36
    2 READ(5,100) (ISIG(J,I),I=1,6),(KSIG(J,I),I=1,6)
    1 IF(I1-7) 3,7,7
    7 DO 5 J=1,36
      ID(J)=0
      DO 4 I=1,6
      ID1=KSIG(J,I)-JSIG(I)
    4 ID(J)=ID(J)+FAC(I)*ID1**2
    5 CONTINUE
      GO TO 17
    3 DO 15 J=1,36
      ID(J)=0
      DO 10 I=1,6
      ID1=ISIG(J,I)-JSIG(I)
   10 ID(J)=ID(J)+FAC(I)*ID1**2
   15 CONTINUE
   17 IS=ID(1)
      DO 20 I=2,36
      IF(ID(J)-IS) 21,21,25
   21 IF(I1-7) 22,8,8
    8 WRITE(6,101) J,(KSIG(J,IJ),IJ=1,6)
      GO TO 25
   22 WRITE(6,101) J,(ISIG(J,IJ),IJ=1,6)
   25 CONTINUE
  100 FORMAT(6I2,I6,5I2)
  101 FORMAT(5X,'A POSSIBLE SIGNATURE IS:',I3,5X,6I2)
  102 FORMAT(1H )
      WRITE(6,102)
      RETURN
      END
```

```
C   MAIN PROGRAM FOR USE WITH SUBROUTINE DET    (FAC NOT INCLU
      DIMENSION ISIG(12,9,6),JSIG(6),LSIG(6)
      DO 1 I=1,6
      DO 1 K=1,9
    1 READ(2,100) (ISIG(I,K,L),L=1,6),(ISIG(I+6,K,L),L=1,6)
      DO 4 I=1,6
      DO 4 J=1,9
      DO 3 K=1,6
      JSIG(K)=ISIG(I,J,K)
    3 LSIG(K)=ISIG(I+6,J,K)
      WRITE(3,101) (JSIG(K),K=1,6),(LSIG(K),K=1,6)
    4 CALL DET(I,J,JSIG,LSIG)
      STOP
  100 FORMAT(12I2)
  101 FORMAT(/,5X,'THE INPUTS ARE:'I4,5I2,' - - ',6I2)
      END


      SUBROUTINE DET(I,J,JSIG,LSIG)
      DIMENSION ISIG(36,6),KSIG(36,6),JSIG(6),LSIG(6),ID(36)
      DO 3 K=1,6
      IF(JSIG(K)-5) 13,12,13
   12 IF(LSIG(K)-5) 13,3,13
    3 CONTINUE
      WRITE(3,101)
      RETURN
   13 IF(I-1) 2,23,2
   23 IF(J-1) 2,33,2
   33 DO 1 K=1,36
    1 READ(2,100) (ISIG(K,L),L=1,6),(KSIG(K,L),L=1,6)
    2 DO 4 K=1,36
      ID(K)=0
      LD(K)=0
      DO 4 L=1,6
      ID1=ISIG(K,L)-JSIG(L)
      ID2=KSIG(K,L)-LSIG(L)
      ID(K)=ID(K)+ID1**2
    4 LD(K)=LD(K)+ID2**2
      DO 5 L=1,6
    5 N(L)=0
      IS=ID(1)
      IT=LD(1)
      DO 7 I1=2,36
      IF(LD(I1)-IT) 16,17,17
   16 IT=LD(I1)
   17 IF(ID(I1)-IS) 6,7,7
    6 IS=ID(I1)
    7 CONTINUE
      DO 10 K=1,36
      IX=(K-1)/6+1
      IF( ID(K)-IS) 35,35,36
   35 N(IX)=N(IX)+1
   36 IF(LD(K)-IT) 37,37,10
   37 N(IX)=N(IX)+1
   10 CONTINUE
      DO 15 K=1,6
      IF(N(K)-2) 40,45,45
   40 IF(N(K)-1) 15,46,15
   45 WRITE(3,102) K,N(K)
      GO TO 15
   46 WRITE(3,103) K
   15 CONTINUE
      RETURN
  100 FORMAT(6I2,I6,5I2)
  102 FORMAT('     THE FAILURE IS PROBABLY COMPONENT',I3,I10)
  103 FORMAT('     THE FAILURE IS POSSIBLY COMPONENT',I3)
  101 FORMAT('     THE SIGNATURE IS NOMINAL')
      END
```

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An investigation of using the response of a circuit at selected test frequencies to isolate faulty circuit components is made. A procedure using a sensitivity approach for intelligent selection of test frequencies is developed. The developed procedure is tested and the results are compared with results using conventional procedures. The effect of random, within tolerance variations of nonfaulty components on the results is studied for both conventional and developed procedures.

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